Can Reservoir Storage Be Uneconomically Large?

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Abstract: The effect of reservoir losses on the economically optimal use of reservoir storage capacity is explored theoretically and numerically. We demonstrate that reservoir seepage and evaporation losses lead to a trade-off between the reliability and mean level of water deliveries, and that as the price elasticity of demand increases and variability of inflows decreases, the optimal reservoir storage decreases. Corresponding reductions in the optimal maximum use of reservoir storage capacity as price elasticity of demand increases, variability of inflows decreases, and losses increase are illustrated. The approach helps clarify the role of storage in maximizing economic benefits from consumptive uses.

DOI: 10.1061/(ASCE)0733-9496(2006)132:6(520)

CE Database subject headings: Reservoir operation; Optimization; River basins; Water demand; Water shortage.

Introduction

The problem of reservoir storage to meet water resources objectives has largely been one of estimating the minimum reservoir capacity required to meet a predetermined schedule of deliveries. The maturation of many analytic and numerical techniques [e.g., Loucks et al. (1981)] coincided with the period where capturing water resources was the primary objective. We present a complementary approach, which seeks to identify the set of reservoir releases and hence storage levels that maximize the economic benefits from a predetermined water resource. We focus our attention on large river basins with over-year carryover storage and demonstrate that storage levels that maximize net economic benefits are highly sensitive to the desired reliability of downstream deliveries.

We examine the simple case of a watershed with inflows to a single pre-existing reservoir. Costs of the existing reservoir are entirely sunk, and operating costs are assumed to be negligible. Benefits of water delivery are given by a constant elasticity demand function. Reservoir releases are the decision variable and storage in each period is the state variable. We estimate numerically the set of releases that maximize the economic benefits to a single water user of deliveries from the reservoir. Our major focus then is the sensitivity of the resulting maximum storage level over the planning period to the price elasticity of demand, the variability of inflows, and reservoir losses (e.g., from evaporation.) The approach is useful for policy and planning purposes because it helps clarify the role of reservoir capacity in maximizing economic benefits from consumptive uses.

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Note. Discussion open until April 1, 2007. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this technical note was submitted for review and possible publication on November 23, 2004; approved on November 17, 2005. This technical note is part of the ***Journal of Water Resources Planning and Management***, Vol. 132, No. 6, November 1, 2006. ©ASCE, ISSN 0733-9496/2006/6-520–523/$25.00.

Background

Sizing reservoir storage to meet a predetermined release is a standard problem [e.g., Rippl (1883), Hazen (1914)] to which Dorfman (1965) applied linear programming by minimizing storage capacity subject to a constant release. In other approaches a predetermined reliability for a fixed water delivery is used to define "successful" project operation (Linsley et al. 1982). Moving beyond reliability, Moy et al. (1986) introduce performance measures focusing on maximum shortfalls and the maximum period of shortfalls, which they identify as system vulnerability and resilience, respectively. Several engineering approaches have recognized that the cost of shortages can be explicitly included in the optimization problem [e.g., Klemes (1979)]. Yeh (1985) identifies a typical planning objective as the maximization of total net annual benefits, while Hashimoto et al. (1982) suggest the desirability (and difficulty) of including "risk-related system performance criteria" in a multiobjective analysis to explicitly value reliability.

The economic literature has emphasized explicit consideration of benefits and costs of water resource development [e.g., Eckstein (1958), Howe (1971)]. More recent work adopts an optimization approach to identify first best solutions to water resource management. Applied to reservoir storage, Burness and Quirk (1980) seek an analytic solution to optimal storage for the case of a perfectly elastic but capacity-constrained demand. Much additional work, both analytic and numerical, has addressed water resource management with storage; most typically, however, the focus is groundwater, where losses over time are generally neglected. However, Krulce et al. (1997) examine a coastal aquifer where increased groundwater stock increases leakage to the ocean.

Method

Our approach is to find the greatest economic value from a given hydrologic system. Either a probability distribution or actual river flows can be used, with economic value defined by the utility of a composite water use consuming deliveries from a single pre-existing reservoir; we assume fixed costs are zero and nonbinding reservoir size. Marginal costs are initially assumed zero. Utility in each time period is assumed to increase with water use at a declining rate (i.e., convex, with a downward sloping economic demand for water). An annual time step is assumed, with over-year (carryover) storage being the critical decision variable. Extension to monthly or shorter time steps would require use of autocorrelated streamflows and a time-varying utility function.

With this approach, a fundamental trade-off exists between the benefits of maintaining carryover storage for future use and the water cost of storage due to evaporation (and other) losses. If evaporation is proportional to exposed surface area, any reservoir with nonvertical walls will suffer losses that increase with storage volume. For example, an inverted pyramid with sides R, S and depth H has a familiar relationship between storage volume and area exposed at the surface. The geometry of similar triangles and elementary algebra shows that $A=\sqrt[3]{9RS/H^2}z^{2/3}$ where A is the exposed area and z is the storage volume, so that surface area is roughly proportional to its volume raised to the two-thirds power.

While evaporation losses thus increase as storage increases (though at a decreasing rate), average annual deliveries from storage are bounded by the mean annual streamflow. Thus, after some point the marginal evaporation losses from increasing storage exceed the marginal potential gain in deliveries. For a given reliability, further increases in storage could decrease annual reservoir deliveries due to the increasing evaporation losses. Therefore we conjecture that for a given set of inflows, a unique set of storage levels (which in some years may equal zero) maximize the total utility gained from reservoir releases.

Numerical Model

The decision maker's objective is to maximize the (discounted) sum of convex utilities $U(x_t)$

$$U=\sum_{t=1}^{T}\beta_t U(x_t) \tag{1}$$

of a renewable inflow Q_t over the planning period $[1,T]$ through choice of release x_t. The mass balance equation is

$$s_t=s_{t-1}+Q_t-x_t-l(s_{t-1}) \tag{2}$$

where at any time t: s_t is the storage of the resource; Q_t is the inflow of the resource into the system; x_t is the water delivery from the reservoir; and $l(s_{t-1})$ is the (evaporative and seepage) loss resultant from storage. The discount factor, β_t, is the discount factor in year t commonly given by $\beta_t=(1+r)^{-t}$, where r is the real discount rate. The model is completed with specification of beginning and ending storage quantities, $s_1=\overline{s_1}$ and $s_T=\overline{s_T}$. The first-order condition for an interior optimal solution is then

$$\frac{MU(x_{t+1})}{MU(x_t)}=\frac{(1+r)}{\left(1-\dfrac{\partial l(s_t)}{\partial s_t}\right)} \tag{3}$$

where the marginal utility $MU(x_t)=\partial U/\partial x_t$. Eq. (3) provides the basic result that current water storage must earn a royalty equal to the discount rate r multiplied by a contribution compensating for system losses. In the case of lossless storage, $\partial l(s_t)/\partial s_t=0$ and Eq. (3) reduces to the well-known Hotelling (1931) result.

Model Eqs. (1) and (2) are solved numerically. For simplicity of discussion, we assume constant price elasticity of demand η leading to

$$U(x_t)=\left(\frac{\eta+1}{\eta}\right)x^{(\eta+1/\eta)} \quad \text{for } \eta\neq-1$$

$$=\ln(x_t) \quad \eta=-1 \tag{4}$$

GAMS (Brooke et al. 1998) and the MINOS nonlinear optimization algorithm (Murtagh and Saunders 1980) numerically solved the nonlinear problem. Default MINOS solution parameters are used, though careful selection of variable upper and lower bounds and attention to starting values are needed for the most nonlinear cases (e.g., price elasticity of demand=−0.1).

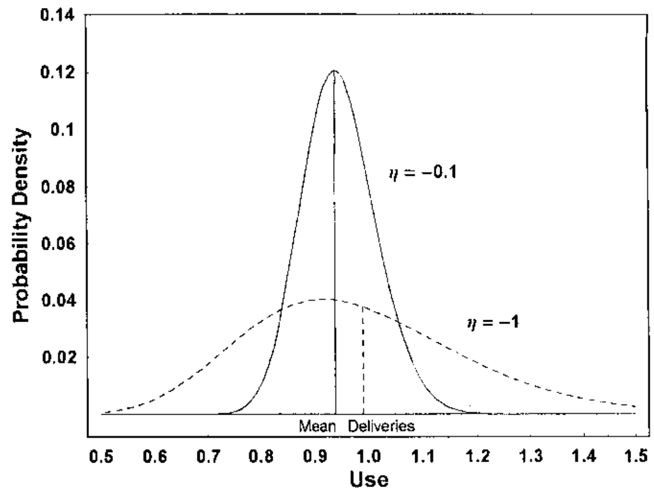

Fig. 1. Trade-off between reliability (determined by price elasticity of demand η) and mean deliveries for 200 period lognormally distributed inflow sequence with mean=1 and standard deviation=0.3. Evaporative losses are 10% per period at storage level equal to mean annual flow.

Sensitivity of Economically Optimal Reservoir Storage

Reservoir storage increases reliability in water-use deliveries, but at the cost of a reduced mean delivery due to evaporation losses (D. Luecke, personal communication, Boulder, Colo. 2000). Fig. 1 shows the trade-off by contrasting the probability density of optimal deliveries estimated by solving Eqs. (1) and (2) with the mean delivery. A 200-year planning period is used, with lognormally distributed inflows for which mean=1 and standard deviation=0.3. Evaporative losses are $l_t=0.10s_t^{2/3}$, and price elasticities of demand of −0.1 and −1.0 are used. The two density functions in Fig. 1 are lognormal regressions of the 200 optimal deliveries for each elasticity and illustrate the trade-off between optimal reliability and mean deliveries for a given reservoir loss function.

In this 200-year planning period, an optimal level of reservoir storage for each year exists based on the economically optimal delivery schedule. Fig. 2 shows the increase in the maximum of these storage levels as the standard deviation of inflows increases, and the decrease in optimal maximum storage as the flexibility of demand increases [defined by an increasing (in magnitude) price elasticity of demand]. The extreme of $\eta=-0.1$ approaches the case of a fixed level of use, while $\eta=-2.0$ indicates a great deal of flexibility in levels of water use. While elasticity estimates are both difficult and have meanings open to interpretation, Dalhuisen

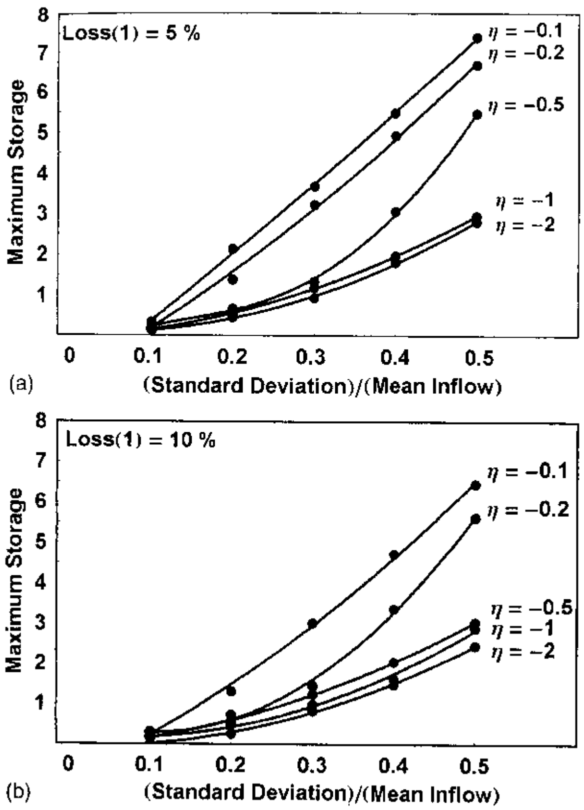

Fig. 2. Optimal reservoir storage as function of variability of inflows: (a) losses are 5% per period when storage equals mean annual flow; at storage levels equal to 2 and 5 times annual flow, evaporation rates are 4%, and 3%, respectively; (b) losses are 10% per period when storage levels equal mean annual flow; at storage levels equal to 2 and 5 times annual flow, evaporation rates are 8 and 6%, respectively

et al. (2001) identify typical elasticities in municipal uses of −0.1 (short run, indoor use) to −0.5 (long run, outdoor use), while Scheierling et al. (2006) estimate elasticities for agricultural uses ranging from inelastic to elastic.

Figs. 2(a and b) contrast maximum storage levels for distinct evaporation rates. Levels of 5 and 10% per period at a level of storage equal to annual flow are used. At storage levels equal to two and five times annual flow, evaporation rates are 20 and 40% lower, respectively. (These are modest evaporation rates; for example, the major Rio Grande reservoir in New Mexico has substantially higher rates.) For our 200-year inflow sequence, relatively elastic demand leads to maximum storage levels of up to about three times annual flow as the standard deviation of inflows increases to 0.5. Very inelastic demands ($\eta=-0.1$) lead to much higher estimated maximum storage levels, while storage in the intermediate and most economically relevant cases ($\eta=-0.2$ to -0.5) is highly sensitive to evaporation rates and variability of inflows.

The exclusion of operating costs may in many cases be unrealistic. High storage levels may for example reduce flood control benefits and inundate recreational areas. Starting from the $\eta=-0.5$ curve in Fig. 2(a), consider the addition of linearly increasing marginal costs of storage equal to 1 and 10% of benefits when storage $s=1$ and water use $x=1$. With costs = 1%, maximum storage levels are 49% (inflow standard deviation = 0.5) to 89% (inflow standard deviation = 0.3) of the levels in the figure, while the respective maximum storage levels with costs = 10% are only 17 to 39% of those in the figure.

Additional Considerations

While it is tempting to suggest that reservoir capacity greater than estimated maximum storage levels is unlikely to be used, additional considerations such as intrayear storage, flood control, hydropower, and recreation demands are typically present. Other modeling simplicities urge caution: for example, insight into the timing and spatial impacts of incremental changes in river basin reservoir capacity are limited with our approach. Finally, the model assumes perfect foresight; in reality, strategies such as hedging would be necessary to cope with limited future knowledge.

Conclusions

We demonstrate that optimal reservoir storage is sensitive to reservoir evaporation and seepage losses, especially when the price elasticity of demand is high and the variability of inflows is low. Under these conditions it is economically beneficial to substitute some reliability in water delivery for larger mean deliveries; the additional deliveries are derived from reductions in reservoir losses made possible by maintaining lower storage levels. We show numerically that as price elasticity of demand increases and variability of inflows decreases, the optimal maximum use of reservoir storage also decreases.

Acknowledgments

The writers would like to thank the reviewers and coeditors for their thorough and important contributions to this note, and we appreciate the helpful comments on previous versions of the manuscript by James Roumasset, Robert A. Young, and John Rowse. All errors remain the responsibility of the writers.

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