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Quantifying the relative importance of flow regulation and grain size regulation of suspended sediment transport α and tracking changes in grain size of bed sediment β

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Abstract. To predict changes in sediment transport, it is essential to know whether transport is regulated mainly by changes in flow or by changes in grain size of sediment on the bed. In flows where changes in suspended sediment transport are regulated purely by changes in flow (grain size of bed sediment is constant), increases in flow strength cause increases in both concentration and grain size of sediment in suspension (because stronger flows are able to suspend more sediment and coarser grains). Under this constraint of constant grain size of bed sediment concentration and median diameter of suspended sediment are positively correlated. In contrast, where transport is regulated purely by changes in grain size of sediment on the bed, concentration and median diameter of suspended sediment are negatively correlated (because increasing the median diameter of the bed sediment causes the concentration to decrease while causing the median grain size in suspension to increase). Where both flow strength and grain size on the bed are free to vary, the relation between concentration and grain size in suspension can be used to quantify the importance of grain size regulation relative to flow regulation of sediment transport, a measure defined as α. To predict sediment transport in systems that are regulated dominantly by changes in grain size on the bed, it is more useful to measure sediment input events or changes in grain size on the bed than to measure changes in flow. More commonly, grain size of bed sediment may be secondary to flow in regulating transport but may, nevertheless, be important. The relative coarseness of bed sediment (β) can be measured directly or, like α, can be calculated from measurements of concentration and grain size of suspended sediment.

1. Introduction

1.1. Background and Purpose

Despite considerable study for nearly a century, predictions of the rate of sediment transport are notoriously inaccurate, particularly for flows outside the laboratory. Predictions are inaccurate not only because the governing processes are not completely understood but also because boundary conditions vary from one site to another or vary through time at a given site. One particularly important boundary condition is grain size of bed sediment. In flows where the grain size of bed sediment changes in response to a changing balance between sediment input and sediment export, the evolving grain size of bed sediment can be the most important factor regulating sediment transport.

It might be argued that the key question to ask when beginning an investigation of a natural sediment-transporting flow is whether transport is limited mainly by flow strength or sediment supply. The answer to this question determines whether research should focus on the relation between flow strength and sediment transport, the rate at which sediment of different grain sizes is supplied to the flow, or both. In this paper we present techniques that can be used to evaluate the importance of changes in flow strength relative to changes in sediment supply in regulating the rate of sediment transport. The aim is not to present a new suspended sediment transport model but rather to use existing models (or equations approximated from existing models) to allow a researcher to make a preliminary evaluation of the relative importance of changing grain size on the bed and changing flow strength in regulating sediment transport in a particular flow. This preliminary evaluation can then be used to guide additional research. Specifically, if changes in sediment transport are regulated mainly by flow, then a measure of the flow strength (e.g., the boundary shear stress, shear velocity, or discharge of water) may be an adequate predictor of sediment transport. In contrast, if changes in sediment transport are regulated mainly by changes in grain size of bed sediment, then measurements of sediment input will be a more accurate predictor of sediment transport than any measure of flow strength. Monitoring of grain size of bed sediment for predictive purposes can be accomplished directly (by extensive bed sampling) or, as described in this paper, can be calculated from concentration and grain size of suspended sediment.

1.2. Definitions

1.2.1. Flow-regulated transport and bed-sediment–regulated transport. In a system where flow-induced changes in transport are large relative to bed sediment grain-size–

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induced changes in transport, transport is defined to be flow-regulated. At the other extreme, where changes in bed sediment grain size are the dominant factor regulating sediment transport, transport is defined to be grain-size–regulated.

1.2.2. Suspended sediment, suspended bed material, and wash load. Suspended sediment includes two kinds of load: suspended bed material and wash load. In this paper the term suspended sediment is applied to suspended bed material (thus excluding wash load). Suspended bed material includes those grain sizes that occur in substantial amounts in the bed, whereas wash load is finer than the bed sediment [*Einstein and Chien*, 1953]. Another approach, compatible with *Einstein and Chien* [1953], might be to base definitions on the concentration gradient; wash load would include those sizes having a concentration that remains constant with height above the bed.

1.2.3. Dependent and independent variables. In this paper, concentration and grain size of suspended sediment are treated as dependent variables; flow strength and grain size of bed sediment are treated as independent variables. It might be argued to the contrary that grain size of bed sediment should be considered a dependent variable because it evolves as a function of flow, sediment input, initial grain size on the bed, and sediment mixing rates. As shown diagrammatically by *Einstein and Chien* [1953], however, grain size on the bed has a "strong and immediate" influence on the transport rate of bed sediment, whereas flow and sediment transport have "weak and slow" influences on grain size of bed sediment. Because changes in grain size have a strong and immediate influence on transport, treating grain size of bed sediment as a time-varying independent variable is physically justified. It is also convenient for the purposes of this paper.

1.3. Previous Work

Beginning early this century, field and lab observations, as well as theory, have shown that the sediment transport rate depends on both grain size and flow strength. As early as 1914, G. K. Gilbert performed laboratory experiments showing that the rate of sediment transport increased with increasing water discharge and decreased with grain size. Since then, there has been a disparity in how evolving grain size has been treated. There have been a number of insightful treatments of interactions between flow and grain size on the bed [*Einstein and Chien*, 1953; *Parker and Klingeman*, 1982; *Proffitt and Sutherland*, 1983; *Parker*, 1990; *Wilcock and McArdell*, 1993]. In general, this previous work has focused on how the bed evolves in response to transport. In the present paper we are primarily interested in how the winnowed bed in turn influences transport.

The relative merits of sediment-recirculating and sediment feed flumes have been debated in detail in the hydrologic literature [e.g., *Parker and Wilcock*, 1993]. Their differing characteristics can be used to illustrate flow-regulated and grain-size–regulated transport. A sediment feed system operated with constant flow but episodic sediment input of bed sediment (with alternating intervals of winnowing and enrichment of fines on the bed) exhibits purely grain-size–regulated transport, whereas a sediment-recirculating flume operated with constant bed sediment but varying discharge exhibits transport that is purely flow-regulated (once the grain size distribution of the actively transported surface layer of bed material becomes established).

Despite an understanding of the importance of bed sediment grain size in regulating suspended sediment transport most hydrological research has centered on how flow regulates sediment transport. Many studies have focused on calculating sediment rating curves, curves that relate sediment transport to water discharge for a particular river. Use of such rating curves implies that while bed sediment variations are important from one river to another, changes through time in grain size on the bed in a specific river are unimportant. Evidently, it is assumed that flow regulation is large relative to grain size regulation (or that changes in grain size on the bed are a function of changes in flow). One goal of this paper is to demonstrate that the opposite can be true: in some flows, changes in grain size on the bed are more important than changes in flow in regulating sediment transport.

In general, development of sediment transport models has focused on refining the flow parameterization, while ignoring evolution of grain size on the bed. Similarly, data collection routinely focuses on time series measurements of flow strength, while changes in bed material are often neglected. There are some notable exceptions to this generalization. *Einstein and Chien* [1953] documented the importance of evolving grain size in regulating sediment transport, *Nordin and Beverage* [1965] demonstrated that sediment transport on the Rio Grande was influenced by evolving bed sediment, *Bennett and Nordin* [1977] developed and tested a model for coupled changes in sediment transport and grain size on the bed produced by changes in the upstream supply of sediment, and several recent studies have included the effects of evolving bed sediment on sediment deposition and resuspension on the continental shelf [*Kachel and Smith*, 1989; *Wiberg et al.*, 1994; *Harris and Wiberg*, 2000]. *Walling and Moorehead* [1989] compiled examples illustrating a variety of ways in which suspended sediment grain size can vary through time in response to changes in discharge and sediment supply. *Reid et al.* [1997] also noted the importance of grain size ("supply control") in regulating suspended sediment transport.

2. Techniques

2.1. Preview: Simple Graphical Clue

Laboratory flumes that recirculate both sediment and water are ideal for studying flow-regulated transport because grain size on the bed remains nearly constant. Under such conditions, increases in shear velocity u_* from one experiment to another cause increases in concentration (Figure 1a), grain size D_s (Figure 1b), and sediment transport q. D_s and C increase because stronger flows are able to suspend coarser sediment and more sediment, and q increases for two reasons: concentrations are higher, and more water is discharged. Because C and D_s increase with shear velocity, they are positively correlated (Figure 1c).

Transport regulated by grain size of bed sediment can be studied by comparing data collected with differing grain sizes of bed sediment for a narrow range of u_*. Under such conditions, coarsening of the bed sediment causes concentration to decrease (Figure 2a), while causing the median diameter of the suspended sediment to increase (Figure 2b). As a result of these opposite responses to changes in D_b, C is inversely related to D_s where transport is regulated by grain size.

If transport in all flows were regulated purely by changes in flow or by changes in bed sediment, then the sign of $\Delta C/\Delta D_s$ would be a definitive distinguishing characteristic (positive where transport is regulated by flow and negative where transport is regulated by grain size. Because all intermediate conditions are possible, however, definitive evaluation is more complicated. Most of this paper is directed toward that more complicated goal: using measured values of $\Delta C/\Delta D_s$ to quantify the relative importance of changes in flow and changes in bed sediment in regulating suspended sediment transport.

2.2. Outline of Approach

Most models of suspended sediment transport express transport q as a function of some combination of flow properties (such as u_*, slope, and depth) and bed sediment properties (such as median diameter D_b and standard deviation). The simplest approach to quantifying the relative importance of a single change in both flow and bed sediment texture is to evaluate their individual impacts on the transport rate. For such a change this measure α can be defined as

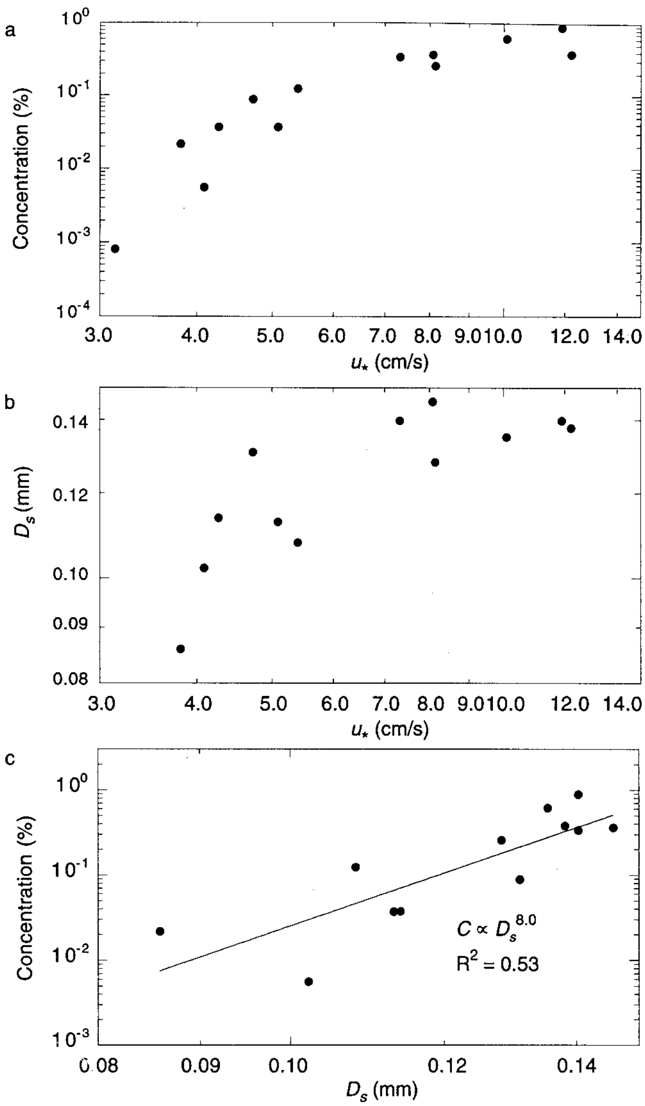

Figure 1. Relations between concentration and grain size for flow-regulated transport: (a) concentration (weight percent) of suspended sediment, (b) median diameter of suspended sediment, D_s, and (c) concentration and D_s are positively correlated because both increase with u_*. For this flow-regulated transport, $C \propto D_s^{8.0}$. Data are from laboratory experiments of *Guy et al.* [1966, Table 9]; for all runs, depth was 15–16 cm; sand in flume had a median diameter of 0.33 mm and sigma phi of 1.04.

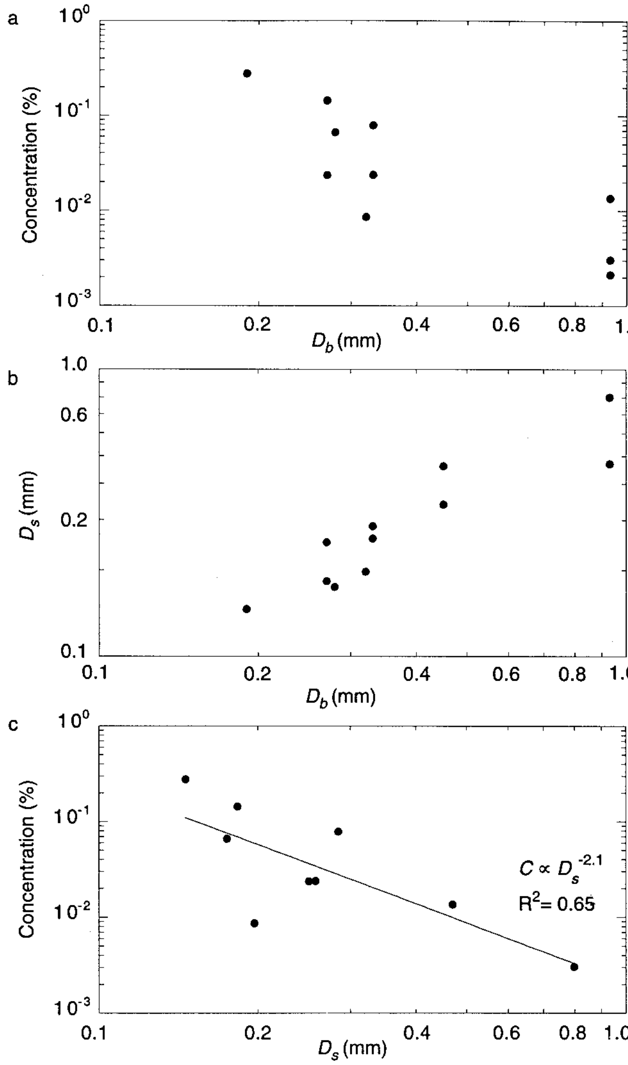

Figure 2. Relations between concentration and grain size for grain-size–regulated transport in flume experiments: (a) concentration (volume percent) of suspended sand decreases as bed sediment coarsens, (b) median diameter of suspended sediment increases as the bed coarsens, and (c) concentration and grain size negatively correlated. In this example, $C \propto D_s^{-2.2}$. Plotted points represent all runs with u_* between 7.0 and 8.0 cm s^{-1} in the data of *Guy et al.* [1966, Tables 2–8]. Other subsets of data selected for narrow ranges of u_* exhibit the same trends as in Figures 2a–2c, but this subset has less scatter than average.

$$\alpha = \frac{\log\left[q(\text{flow}_1, \text{grain size}_1)/q(\text{flow}_1, \text{grain size}_2)\right]}{\log\left[q(\text{flow}_1, \text{grain size}_1)/q(\text{flow}_2, \text{grain size}_1)\right]}, \qquad (1)$$

where q gives the sediment transport rate as a function of both flow and bed sediment grain size; subscripts refer to conditions at two times. The numerator quantifies the extent to which a change in transport rate is influenced by the change in bed sediment grain size (holding flow constant), while the denominator quantifies the effect of the change in flow alone. Here α is a dimensionless number that describes how much of a change in transport is caused by a change in bed sediment relative to a change in flow. Where sediment transport is regulated primarily by changes in grain size on the bed, $|\alpha| \gg 1$; where transport is regulated primarily by changes in flow, $|\alpha|$

$\ll 1$; and where transport is regulated equally by changes in flow and bed sediment, $|\alpha| = 1$. As employed below, (1) is useful because of three properties: the numerator is independent of flow, the denominator is independent of grain size, and log (q) is a linear function of log (flow) and log (grain size). These properties all follow from (3), which is introduced below.

Although the approach outlined above is valid theoretically, it would be impractical to implement an adequate bed sediment sampling program. First, some particles on the bed are too coarse to be moved by the flow at the time of sampling and should not be used to evaluate D_b; this problem is complicated because the grain size threshold of transportable particles varies with flow strength. Second, grain size commonly varies beneath the sediment surface, and a person collecting samples does not know how deep to sample or may be unable to restrict a sample to the surface, a problem noted by *Einstein and Chien* [1953]. Third, grain size commonly varies spatially, requiring a large number of samples for results to be representative. Similarly, it is often difficult to collect representative flow data in a spatially varying flow. To eliminate these sampling difficulties, it is preferable to invert the problem and use easily measurable dependent variables (concentration of suspended sediment C and median diameter of suspended sediment D_s) to solve for the independent variables. This approach eliminates the sampling problems listed above since only particles capable of being transported are sampled and the inherent mixing of suspended sediment reduces the number of samples that are required to be representative.

The goal then becomes to derive a set of equations that express changes in the hard-to-evaluate independent variables (flow and grain size of bed sediment) as functions of the more easily observable dependent variables (concentration and grain size of suspended sediment). Calculated values of the independent variables can then be substituted in a specific sediment transport relation represented conceptually in (1). Several approaches can be taken to derive appropriate equations. One approach might be to begin with simple transport equations that could be solved algebraically. Another approach would be to use a more detailed numerical model [e.g., *McLean*, 1992] that calculates the concentration of every grain size at many elevations in the flow. By integrating the equations in such a model it might be possible to derive an analytical solution. A third approach would be to use a model like McLean's to predict the dependent variables from the independent variables for a wide range of representative conditions. The calculated results could then be approximated with simpler mathematical functions to simplify the problem of inverting the equations. In section 2.3 we follow this approach.

Determining the relative importance of flow-regulated and grain-size–regulated transport is relatively insensitive to the particular transport equations used. For example, incorporating dune development in the suspended sediment transport algorithm causes moderately large changes in the predicted values of concentration and median diameter of sediment in suspension but has a much smaller effect on the predicted importance of grain size regulation relative to flow regulation. Incorporating the effects of density stratification due to high concentration gradients of suspended sediment has an even smaller effect.

2.3. Quantifying the Relative Importance of Grain Size Regulation and Flow Regulation of Suspended Sediment Transport

To quantify the relative influence of flow and bed sediment in regulating sediment transport, we used the following approach: (1) a numerical model based on *McLean* [1992] was used to calculate concentration of suspended sediment at 500 logarithmically spaced elevations above the bed for 129 size classes of bed sediment binned in 1/16 ϕ increments; details of the model are described in Appendix A. The algorithm was used to predict mean concentration and median grain diameter for more than 1000 combinations of flow variables, including 11 median grain diameters (0.03–1.2 mm), 20 values of u_* (from below threshold of transport to upper plane bed regime), 3 depths (10, 100, and 1000 cm), and both narrow and wide lognormal bed sediment grain size distributions. The two size distributions had values of sigma phi of 0.55 and 1.4 (68% of the grains had diameters within a factor of $2^{0.55}$ or $2^{1.4}$ of the median diameter). These values of sigma phi bracket the size distributions typical of most rivers and continental shelves [*Friedman et al.*, 1992]. The computations were repeated for a more complex algorithm that included development of dunes on the bed. (2) Concentration and D_s were averaged through the water column (Figure 3a). (3) The computed results were then approximated by equations expressing concentration and median diameter of suspended sediment as power functions of u_* and D_b. The approximations fit the computed results quite well ($R^2 = 0.93–0.95$ for the no-dune model) for conditions ranging from near-threshold to suspended load layer reference concentrations of 50% and for bed sediment Rouse numbers of 0.02–1. Including dunes in the model resulted in a poorer fit of the power law approximations (values of R^2 were reduced to 0.5–0.9) but has little effect on the results (α and β, defined below).

This sequence of computations led to

$$C \propto u_*^J D_b^K, \tag{2}$$

where the values of J and K for various models are given in Table 1. Equation (2) is similar to one derived by *Engelund and Hansen* [1967] for calculating total sediment load; holding fluid and sediment densities constant and assuming a constant friction factor, their equation (4.3.5) reduces to $C \propto u_*^4 D_b^{-1}$.

Making the approximations that for a given channel geometry $q \propto CU$ and that $U \propto u_*$ leads to

$$q \propto u_*^{J+1} D_b^K. \tag{3}$$

The relative influence of u_* and D_b on regulating the rate of sediment transport depends not only on $J + 1$ relative to K but also on how much u_* and D_b change through time in a particular sediment transport system. Quantifying the relative magnitudes of Δu_*^{J+1} and ΔD_b^K for a particular river would be relatively simple if u_* and D_b were known or could be easily measured in the field. The relation generalized in (1) could be approximated by

$$\alpha = \left(\frac{K}{J+1}\right)\left(\frac{\log \Delta D_b}{\log \Delta u_*}\right), \tag{4}$$

where Δ signifies the ratio of two values of a variable measured at two different times.

As mentioned above, however, u_* and D_b are often unknown. It is therefore more convenient to invert the problem so that u_* and D_b can be determined from C and D_s. Equation (5) was derived for this purpose by approximating the computational results with

$$D_s \propto u_*^L D_b^M. \tag{5}$$

L and M both vary with grain size distribution of the bed sediment. In the limit where all grains have the same diameter, L equals 0, and M equals 1. At the other extreme, for an extremely broad size distribution (broader than sigma phi = 1.4 used in our wide-distribution computations), D_s could be expected to vary primarily with flow strength rather than with median diameter of the bed sediment ($M \approx 0$). For more realistic intermediate conditions (sigma phi = 0.55 and sigma phi = 1.4), values of L and M are determined by approximating the computational results (Table 1).

To express the independent variables (Δu_* and ΔD_b) in terms of the dependent variables (C and D_s), we can simultaneously solve (2) and (5):

$$\Delta D_b = (\Delta C)^{\frac{-L}{JM-KL}}(\Delta D_s)^{\frac{J}{JM-KL}} \tag{6}$$

$$\Delta u_* = (\Delta C)^{\frac{M}{JM-KL}}(\Delta D_s)^{\frac{-K}{JM-KL}}. \tag{7}$$

As in (4), Δ signifies the ratio of values of a variable at two different times. The values of ΔD_b and Δu_* calculated in (6)–(7) are then used to evaluate α in (4).

The technique described above can be summarized in two simple steps. First, changes in the dependent variables C and D_s are used to calculate changes in the independent variables u_* and D_b (equations (6) and (7)). Second, the change in transport resulting from the calculated Δu_* and ΔD_b is quantified (equation (4)). Combining the algebraic steps in (4), (6), and (7) into a single equation yields

$$\alpha = \left(\frac{K}{J+1}\right)\frac{-L\left(\frac{\log \Delta C}{\log \Delta D_s}\right)+J}{M\left(\frac{\log \Delta C}{\log \Delta D_s}\right)-K}. \tag{8}$$

The algebraic operations represented by (4), (6), and (7) can be visualized as three graphical operations in Figure 3b. First, a vector is plotted, with endpoints defined by the observed initial and final values of C and D_s (plotted relative to the contoured values of these dependent variables). Then ΔD_b and Δu_* are determined by measuring the projections of this vector on the x and y axes. Finally, the two changes in q resulting from changes in u_* and D_b are evaluated from the contoured values of q.

The symbol Δ in (1), (4), and (6)–(8) applies to a single change in conditions; for results to be representative of a more extensive set of time series measurements, it is necessary to replace the Δ with a statistical measure. The approach taken here is to replace Δ with the standard deviation of a variable so that $\log \Delta C/\log \Delta D_s$ is replaced by

$$\left|\frac{\log \Delta C}{\log \Delta D_s}\right| \approx \frac{\sigma(\log C)}{\sigma(\log D_s)}, \tag{9}$$

where $\sigma(\log C)$ and $\sigma(\log D_s)$ represent the standard deviations of $\log C$ and $\log D_s$, respectively. This approach of substituting the standard deviation of a variable for a single change Δ in that variable is equivalent to the reduced major axis technique for fitting a line to a scatterplot of x–y data

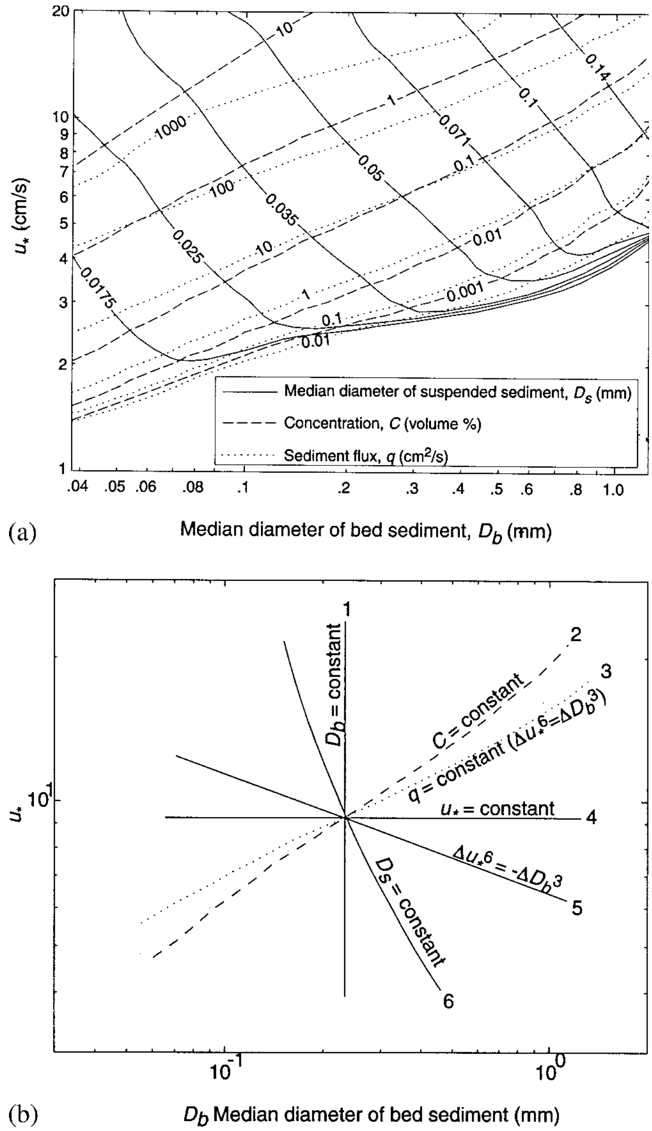

Figure 3. Contours of calculated suspended sediment median diameter D_s, concentration C, and transport rate q, plotted as a function of shear velocity u_* and median diameter of bed sediment D_b: (a) results for a wide range of u_* and D_b and (b) enlargement showing representative possible changes in independent and dependent variables. Bed material was assigned a lognormal distribution having sigma phi = 1.4 (68% of the grains had diameters within a factor of $2^{1.4}$ of the median diameter). Calculations are for a depth of 100 cm; results for depths of 10 and 1000 cm are similar. The center point represents initial conditions; six hypothetical changes are represented by numbers 1–6 corresponding to constant critical values in Table 2.

[*Davis*, 1986, pp. 200–204]. Equation (9) predicts the absolute value of $\log \Delta C/\log \Delta D_s$ but not the sign of this quantity. In some cases the sign can be determined by inspection; in other cases it may be necessary to determine whether the positive or negative sign gives a better fit to the data [*Davis*, 1986].

In summary, this technique involves three steps: the standard deviations of $\log C$ and $\log D_s$ are used to determine $\log \Delta C/\log \Delta D_s$ (equation (9)); ΔC and ΔD_s are used to solve for Δu_* and ΔD_b (equations (6) and (7)), and Δu_* and ΔD_b are used to solve for α (equation (4)). Equation (8) combines the

Table 1. Values of Exponents in Equations (2)–(8) and (11)[a]

Model	J	K	L	M	$M/(JM - KL)$	$-K/(JM - KL)$	$-L/(JM - KL)$	$J/(JM - KL)$
Without dunes; sigma phi = 0.55	3.5	−2.5	0.15	1.0	0.90	0.65	−0.039	0.26
Without dunes; sigma phi = 1.4	3.5	−1.5	0.4	0.5	1.5	0.64	−0.17	0.21
With dunes; sigma phi = 0.55	5.0	−3.0	0.2	0.7	1.2	0.73	−0.05	0.17
With dunes; sigma phi = 1.4	3.5	−1.5	0.3	0.5	1.5	0.66	−0.15	0.22

[a]Exponents were evaluated by fitting power laws to computational results such as those plotted in Figure 3.

operations of these last two steps. Alternatively, log ΔC/log ΔD_s determined from (9) could be used to determine α graphically, as discussed in section 2.5.

For improved accuracy it may prove useful to restrict data used to calculate α to times when sediment transport is substantial or to weight measurements by the transport rate (or concentration) at the time of the measurement. The latter approach was used with data from rivers presented in section 3.

2.4. Tracking Grain Size of Bed Sediment That is Accessible to the Flow

Once it has been established that grain size regulation of sediment transport is important ($|\alpha|$ approaches or exceeds 1) in a particular sediment transport system, it may be desirable to monitor changes in grain size of sediment on the bed. This is useful for at least three goals: (1) quantifying changes through time in the degree of winnowing or armoring downstream from a dam, (2) measuring the extent to which tributaries have contributed fine sediment to the bed of a channel (as is important in determining the timing of artificial floods in the Colorado River in Grand Canyon), and (3) measuring the spatial (depth-related) variation of grain size of sediment on the bed in pools, bars, and floodplains.

A dimensionless measure of grain size of sediment on the bed, β, can be defined as

$$\beta = \frac{D_b}{D_{bm}}, \tag{10}$$

where D_b is the median grain diameter of bed sediment at an instant in time and D_{bm} is the average of a sequence of median diameters at the same location. Thus β is a measure of the relative coarseness of sediment on the bed. By substituting β for ΔD_b and expressing ΔC and ΔD_s relative to their mean or median values ΔC_m and ΔD_{sm}, (6) becomes

$$\beta = \left(\frac{C}{C_m}\right)^{\frac{-L}{JM-KL}} \left(\frac{D_s}{D_{sm}}\right)^{\frac{J}{JM-KL}}. \tag{11}$$

The exponent of the concentration ratio is negative, whereas the exponent of the grain size ratio is positive (Table 1). As a result, the relative bed coarseness β increases as concentration decreases and as grain size increases (as intuition would suggest). Bed sediment grain size is proportional to β and can be calculated by multiplying β by D_{bm}, a constant representing the time-averaged bed sediment grain size for a particular reach.

The relation between bed sediment and suspended sediment expressed in (11) reflects at least three kinds of changes. First, grain size of sediment at a point on the bed (or within a reach) can change through time as a result of deposition of sediment from upstream or tributaries, winnowing of the bed, or erosion and excavation of underlying substrate. Second, the depth to which sediment in the substrate interacts with the flow may vary with flow strength [*Wiberg et al.*, 1994]. For example, a weak flow that generates ripples on the bed will exchange sediment with the uppermost few centimeters of the sediment substrate. In contrast, a stronger flow that generates large dunes will exchange sediment with a greater depth within the substrate. Third, as stage increases, a river may gain access to finer sediment that occurs on high-elevation channel margin bars and floodplains. (This stage dependence of source grain size may be important in rivers but is not important on the continental shelf.) Of these three changes, only the first reflects actual changes on the bed; the latter two changes in grain size reflect lateral or vertical changes in the region of the channel interacting with the flow. Measured changes in β reflect all of these factors, as illustrated in section 3.

Where winnowing of bed sediment is extreme, coarsening of the bed may be accompanied by reduction in surface area of fine-grained (transportable) sediment patches on the bed. Calculations of α and β ignore such changes in surface area. Where such areal changes are substantial, the effect of ignoring them will be to introduce error in calculated values of α and β. If a decrease in surface area of fine-sediment patches on the bed causes a reduced concentration of suspended sediment, then part of the calculated bed coarsening β will be due to local removal of fine sediment in addition to coarsening of existing patches of fine sediment. This potential error is relatively small. For example, if changes in the fine-sediment patch area cause a 50% change in concentration of suspended sediment, the resulting error in β is only $1.5^{-L/(JM-KL)}$, or 2–7%, for all exponent values in Table 1.

β has some aspects in common with a measure developed by *Dietrich et al.* [1989]. They proposed that the magnitude of bed armoring could be quantified by q_*, a dimensionless ratio of the transport rate of a coarsened surface layer to the transport rate of the finer substrate. Their measure quantifies the degree to which surficial armoring has reduced transport. Our measure and that of Dietrich et al. are both based on the idea that coarsening of the bed influences transport. The two measures have substantial practical and theoretical differences, however. Instead of addressing the relative transportability of the surface layer and the substrate, our technique evaluates the effect of temporal changes in the bed sediment grain size on the grain size and concentration of suspended sediment. This technique exploits the natural averaging inherent in suspension processes and requires only sampling suspended sediment. This technique is easily applied in flows where the "surficial" bed sediment is mixed by migrating dunes into layers that can exceed a meter or more. It is also applicable where surficial sediment is finer than the substrate.

2.5. Graphical Solutions

The relative importance of changes in flow strength and changes in bed sediment grain size can be determined graphically (Figure 4 and Table 2). As shown in above, an inverse correlation between concentration and grain size of suspended sediment is a clue that grain size regulation of transport may be important. A more precise evaluation of α can be determined from C and D_s using the curves in Figure 4, which were calculated using (8) and the values in Table 1. To use the curves in Figure 4, $\log \Delta C/\log \Delta D_s$ must first be determined. If $\log C$ and $\log D_s$ are highly correlated, then $\log \Delta C/\log \Delta D_s$ can be quantified by a simple regression of $\log C$ against $\log D_s$. Typically, however, $\log C$ and $\log D_s$ will be only weakly correlated, and $\log \Delta C/\log \Delta D_s$ will be evaluated using (9) (remembering to determine the sign of $\log \Delta C/\log \Delta D_s$).

If the equations developed above worked perfectly, and if experimental data were free of error, then points that were plotted using either D_s and C or D_b and u_* in Figure 3a should be coincident. Although tests with experimental data indicate that this technique is only moderately successful at predicting the actual values of the independent variables, the equations are more successful in determining relative changes in u_* and D_b. Evidently, the equations more accurately describe the slopes of the surfaces contoured in Figure 3a than actual values on the surfaces.

In the following examples, α was determined using (8)–(9). The same results can be obtained by determining $\log \Delta C/\log \Delta D_s$ and then reading the corresponding value of α in Figure 4.

3. Examples

3.1. Example 1: Flume Data

In the previous discussion, flume data were used to illustrate flow-regulated transport and grain-size–regulated transport (Figures 1–2) but were not utilized in the computational results or algebraic approximations. It is therefore of interest to see how well (1)–(9) work when tested with flume data. Results of these tests are summarized in Figure 4c and Table 3. For subsets of flume data [*Guy et al.*, 1966] with narrow ranges of grain size (dominantly flow-regulated transport), values of $|\alpha|$

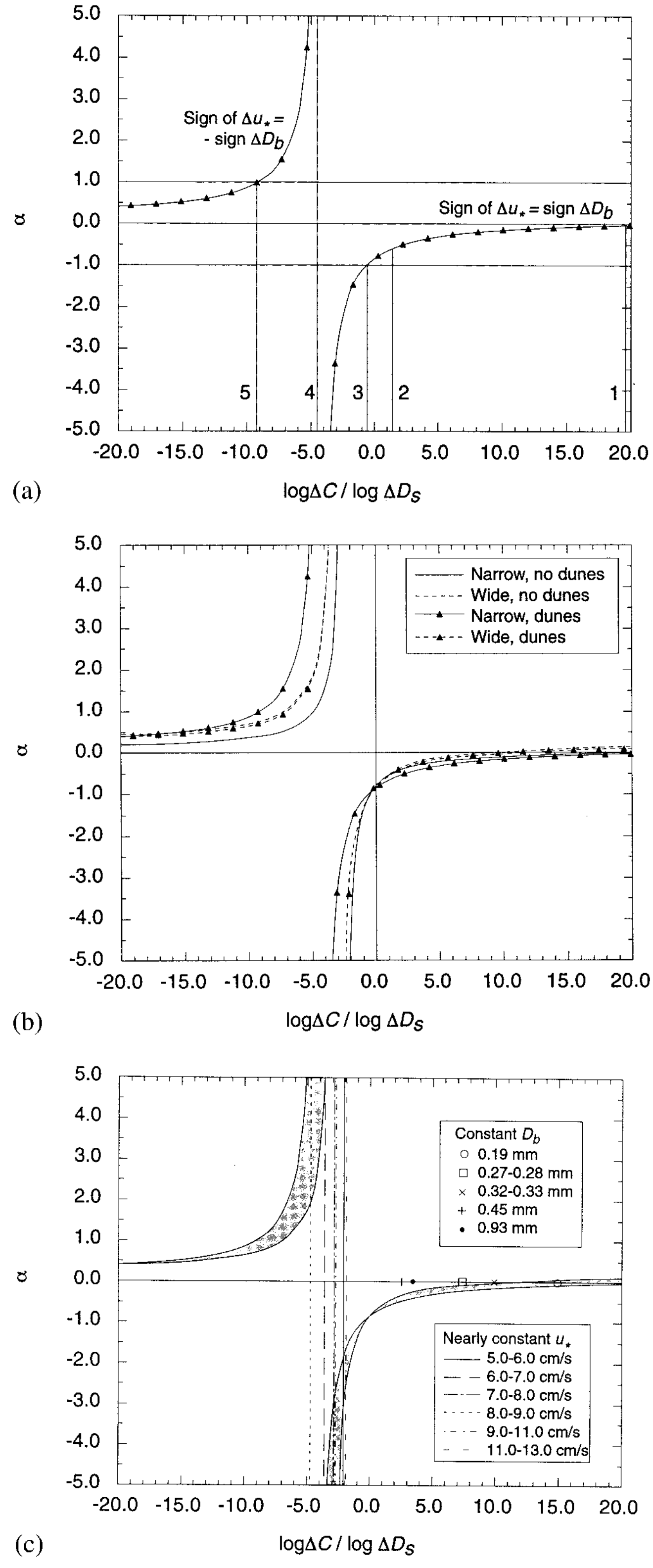

Figure 4. (opposite) Plot of α (solid line) as a function of $\log \Delta C/\log \Delta D_s$. (a) Dashed vertical lines (numbered 1–5) depict the critical cases illustrated in Figure 3b and Tables 2–3. Transport is flow-regulated where $|\alpha| < 1.0$ and grain-size–regulated where $|\alpha| > 1.0$. To use Figure a, $\log \Delta C/\log \Delta D_s$ is determined by fitting a power law to a scatterplot of $\log C$ versus $\log D_s$ using the reduced major axis technique [*Davis*, 1986]. Plot was calculated using a model with narrow grain size distribution (sigma phi = 0.55) and dune development. (b) Plot showing the variability of α for four different models: narrow and wide grain size distributions (sigma phi = 0.55 and 1.4) with and without dune development. (c) Predicted and observed $\log \Delta C/\log \Delta D_s$ for critical values of α. Shaded bands show predicted α (as in Figure 4b). Plotted points show observed values of $\log \Delta C/\log \Delta D_s$ for flume runs with constant bed sediment (flow-regulated transport, $\alpha = 0$). Vertical lines show observed values of $\log \Delta C/\log \Delta D_s$ for flume runs with nearly constant u_* (grain size–regulated transport, $|\alpha| = \infty$).

Table 2. Predicted Changes in Suspended Sediment and α for Cases Specified in Figure 3b[a]

Case	ΔC	ΔD_s	Δq	$\log \Delta C / \log \Delta D_s$	α (calculated using (8))
1	+	+	+	10–25	0.0
D_b = const	−	−	−		(by definition)
2	=	+	+	0.0	
C = const	=	−	−	(by definition)	−0.75 to −0.84
$\Delta U^J \approx -\Delta D_b^k$					
3	−	+	=	−0.83 to −0.54	−1.0
q = const	+	−	=		(by definition)
$\Delta U^{j+1} \approx \Delta D_b^k$					
4	−	+	−	−6 to −3	$-\infty$
u_* = const	+	−	+		(by definition)
5	−	+	−		1.0
$q \neq$ const	+	−	+	−14 to −7	(by definition)
$\Delta U^{J+1} \approx -\Delta D_b^k$					
6	−	=	−	$-\infty$	0.14–0.23
D_s = const	+	=	+	(by definition)	

[a]Some critical values are determined by definition; the others are calculated using two values of sigma phi of bed sediment (0.55 and 1.4).

calculated from the standard deviations of u_* and D_b range from 0.0 to 0.039, and values calculated from standard deviations of C and D_s range from 0.077 to 0.40 (Table 3). For subsets of data with narrow ranges of u_* (dominantly grain-size–regulated transport), values of $|\alpha|$ calculated from u_* and D_b range from 1.8 to 14.5, and values calculated from C and D_s range from 1.6 to 8.1 (Table 3). In all of these cases, calculated values of $|\alpha|$ fall on the appropriate side of the critical value of 1.0 (<1.0 for flow-regulated transport and >1.0 for grain-size–regulated transport).

Einstein and Chien [1953] studied the effects of evolving grain size on sediment transport, and their results are ideal for testing predictions of β. In series C of their experiments they conducted five flume runs at constant discharge and depth. In the first runs (C-1 to C-3) they added fine sediment to the recirculating flow. In the next runs (C-4 and C-5) they stopped adding sediment and started replacing some of the sediment-laden water in the flume with clear water. They measured the grain size of sediment on the bed as well as the two dependent variables necessary to evaluate β in (11). All three variables document the fining of the bed from run C-1 to run C-3 and the coarsening of the bed from run C-3 to run C-5 (Figure 5). The correlation between predicted and observed grain size of bed sediment is high ($R = 0.91$).

Table 3. Observed Values of $\log \Delta C / \log \Delta D_s$ and Corresponding Predicted and Observed Values of α for Flume Data of *Guy et al.* [1966][a]

	Observed α (calculated using standard deviations of u_* and D_b in (4))	$\log \Delta C / \log \Delta D_s$, (calculated using (9))	α (predicted using (8)–(9))
D_b = 0.19 mm	0.0	14.8	−0.077
D_b = 0.27–0.28 mm	0.039	7.5	−0.21
D_b = 0.32–0.33 mm	0.020	10.3	−0.14
D_b = 0.45 mm	0.0	2.6	−0.47
D_b = 0.93 mm	0.0	3.4	−0.40
u_* = 5.0–6.0 cm s^{-1}	−1.8	−2.1	−1.7
u_* = 6.0–7.0 cm s^{-1}	−1.9	−3.7	−6.8
u_* = 7.0–8.0 cm s^{-1}	−7.8	−2.7	−2.4
u_* = 8.0–9.0 cm s^{-1}	−14.5	−4.8	8.1
u_* = 9.0–11.0 cm s^{-1}	−4.3	−2.7	−2.8
u_* = 11.0–13.0 cm s^{-1}	−7.0	−1.9	−1.6

[a]Predicted values of α were calculated using exponents evaluated for dune bed and bed sediment with sigma phi = 0.55.

3.2. Example 2: 1996 Grand Canyon Flood Experiment

The following four examples are based on the Colorado River and one of its tributaries (Paria River) in the Grand Canyon region of Arizona. Glen Canyon Dam was completed on the Colorado River in 1963. The dam was built 26 km upstream from Lees Ferry (and the confluence with the Paria River) and 166 km upstream from the Grand Canyon gage.

An example of grain size regulation of sediment transport occurred during an experimental flood on the Colorado River in Grand Canyon in 1996 [*Rubin et al.*, 1998; *Topping et al.*, 1999]. For the 7 days of the flood experiment, clear water was released from Glen Canyon Dam at the rate of 1270 m^3 s^{-1}. In response to this erosive flow the bed at the Grand Canyon gage coarsened, which caused suspended sediment both to coarsen and to decrease in concentration (Figures 6a–6b). The resulting negative correlation between suspended sediment concentration and grain size (Figure 6c) demonstrates that grain size regulation was important during this event.

Applying the graphical technique for evaluating α shows $\log \Delta C / \log \Delta D_s = -1.6$ (exponent of power law regression line in Figure 6c); the correlation is high, so that this fit is close to the reduced major axis solution ($\log \Delta C / \log \Delta D_s = -1.7$). In Figure 4b we find that the corresponding value of α is −1.5. Analysis of suspended sediment collected at two other sites during the flood yields values of α equal to −6.3 (196 km

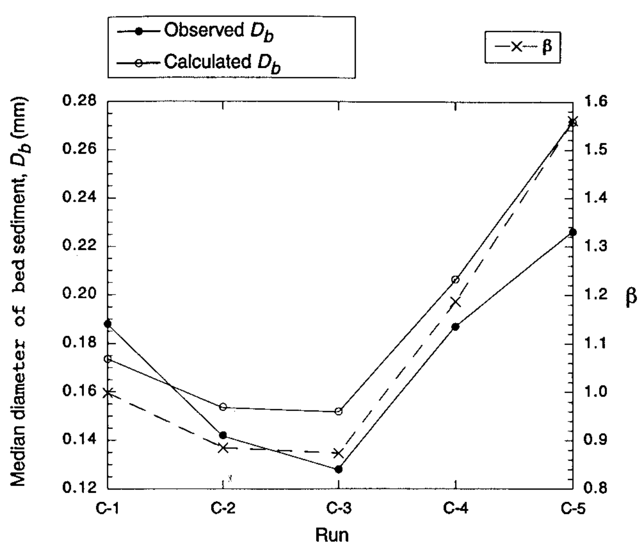

Figure 5. Grain size evolution in runs C-1 to C-5 of *Einstein and Chien* [1953]. Addition of fine sediment to the steady recirculating flow caused the bed to aggrade and fine from C-1 to C-3. Extraction of a portion of the suspended fines from the flow caused the bed to degrade and coarsen from C-3 to C-5. Observed surficial grain size D_b was measured by Einstein and Chien; β was calculated in (11) using the concentration and grain size of suspended sediment; calculated values of D_b were determined by multiplying β by the mean diameter of surficial bed sediment for all five runs. The calculated changes follow the same trends as the observed changes.

downstream from Lees Ferry, at the mouth of Hundred Twenty-Two Mile Creek) and −3.3 (268 km downstream from Lees Ferry, at the National Canyon gage site). All values indicate sediment transport that is regulated by changes in bed sediment grain size ($|\alpha| > 1$).

The same observations of suspended sediment concentration and grain size can be used (in (11)) to calculate changes in the relative coarseness of sediment on the bed during the flood (Figure 7). Comparison with sampled bed sediment not only shows good agreement but the reduced scatter in the calculated values suggests that the calculations may be more representative of the system than the values measured at a single cross section. In this case, where river discharge was constant, changes in β reflect actual changes in grain size of sediment on the bed. In other situations, where discharge is free to vary, calculated changes in β can reflect changes in grain size on the bed, as well as changes in the region of the bed that is accessible to the flow.

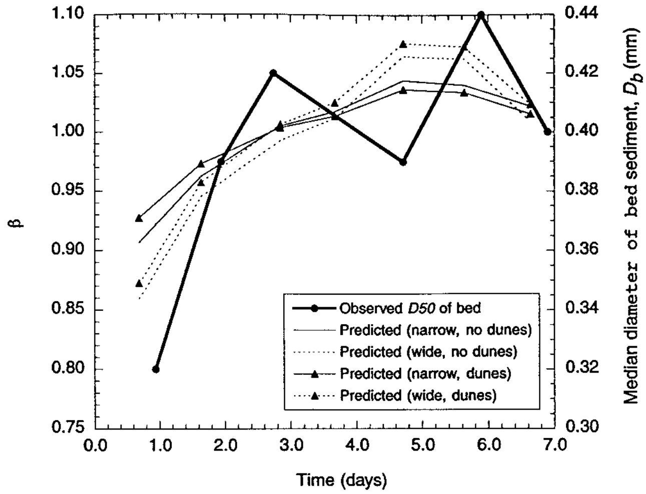

Figure 7. Plot of β and predicted and observed bed sediment median diameter during the 1996 flood experiment. Observed bed sediment median diameter was determined from samples collected at three to five locations at the Grand Canyon gage cableway [*Rubin et al.*, 1998; *Topping et al.*, 1999]; β was calculated using (11) and suspended sediment measurements; predicted values of bed sediment median diameter were calculated by expressing β relative to the median diameter of all bed samples. The predicted values of bed sediment diameter are in close agreement with observed values. The predicted values have less scatter than the values observed at a single cross section and may be more representative of the river.

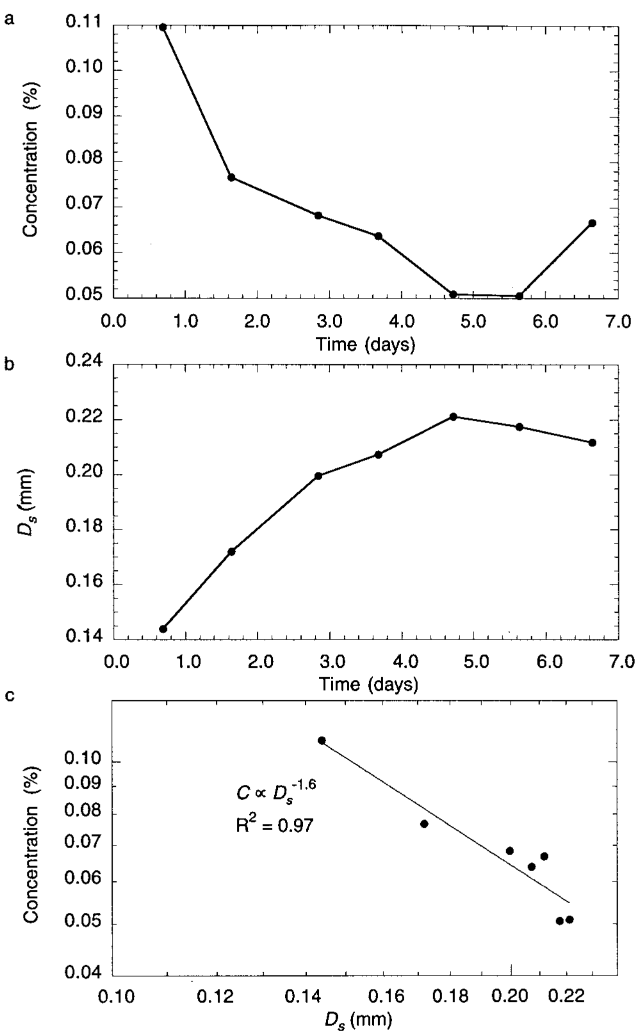

Figure 6. Relations between concentration and grain size for grain-size–regulated transport at Grand Canyon gage during the 1996 flood experiment [*Rubin et al.*, 1998; *Topping et al.*, 1999]: (a) concentration (volume percent) of suspended sand decreased through time, (b) median diameter of suspended sand, D_s, increased through time, and (c) concentration and grain size are negatively correlated ($\alpha = -1.5$). As sediment on the bed was winnowed, suspended sediment decreased in concentration and increased in grain size.

3.3. Example 3: Paria River

The Paria River is the first major tributary to the Colorado River downstream from Glen Canyon Dam. The value of α calculated for the Paria River (1954–1983) is −0.16 (calculated by weighting all suspended sediment measurements equally) or −0.09 (weighting each suspended sediment measurement proportionately to the concentration). Both values are near zero, the theoretical value for flow-regulated transport. Details of the data suggest, however, that the Paria has a stage-dependent variation in grain size of available sediment that complicates this interpretation. As discharge increases, the grain size of sediment on the bed decreases because the higher discharge flows have access to finer sediment along the banks and floodplain [*Topping*, 1997]. A scatterplot of β as a function of discharge illustrates this process (Figure 8a). A lack of hysteresis suggests that the grain size on the bed does not evolve seasonally. Together α and β provide much of the guidance required to model this river: the bed sediment grain size accessible to the river varies with stage but not with season.

3.4. Example 4: Colorado River Before Glen Canyon Dam

Previous work has shown that grain size evolved during snowmelt floods in the predam Colorado River, producing inversely graded flood deposits [*Rubin et al.*, 1998; *Topping et al.*, 1999] and influencing the transport rate seasonally [*Topping et al.*, 2000]. The present analysis suggests that although sediment transport was regulated in part by evolving grain size on the bed, changes in flow were a more important influence on the transport rate because the predam range in flow was large. At the Grand Canyon gage (1944–1962), α = −0.2

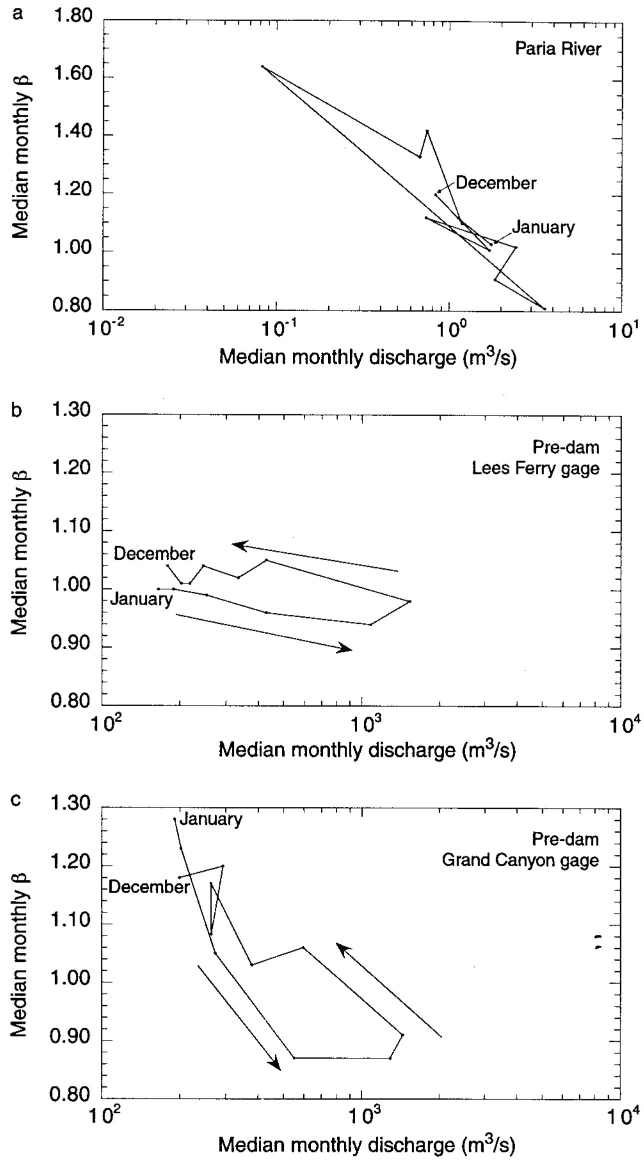

Figure 8. Plots showing relations between mean monthly discharge and mean monthly β at three locations. At all locations the bed fines (β decreases) as stage increases and the flow gains access to finer sediment on the floodplain or along the channel. (a) Paria River at Lees Ferry (1954–1983). This example shows the greatest variation in β as a function of discharge (factor of 2). The data fall along a single line (no hysteresis loop), indicating no detectable systematic grain size evolution. (b) Predam Colorado River at the Lees Ferry gage (1944–1962). Compared to the Paria River, this site shows less variability of bed sediment grain size with discharge (~10%) but does display detectable hysteresis during the snowmelt flood. (c) Predam Colorado River at the Grand Canyon gage (1944–1962). This site shows greater variability of bed sediment grain size with discharge and greater hysteresis than the Lees Ferry gage.

unweighted, and $\alpha = -0.1$ weighted; at the Lees Ferry gage (1949–1962), $\alpha = -0.3$ (both unweighted and weighted).

A comparison of the annual variation in β and discharge indicates that the grain size of bed sediment varied in two ways (Figures 8b–8c). First, finer sediment was more accessible at high stage. Second, for a given stage the bed was finer during the rising limb of the annual flood than during the recession. To model sediment transport in the predam river thus requires both a stage-dependent grain size of bed material and a winnowing of the bed during floods. Both tendencies are more pronounced at the Grand Canyon gage than at the Lees Ferry gage (Figures 8b–8c).

Because the flow was similar at the Lees Ferry and Grand Canyon gages, differences in β between the two sites (Figures 8b–8c) must have arisen from other factors. The greater hysteresis at the Grand Canyon gage indicates that seasonal storage and depletion of fine sediment was more pronounced there than at the Lees Ferry gage, as *Topping et al.* [2000] documented using a different approach. The greater range in values of β at the Grand Canyon gage indicates that grain size of available sediment was more dependent on stage there than at the Lees Ferry gage.

3.5. Example 5: Colorado River After Glen Canyon Dam

In the postdam river at the Grand Canyon gage (1964–1986), bed sediment grain size has been at least as important as flow in regulating sediment transport rate ($|\alpha| = 6.0$ unweighted; $|\alpha| = 1.0$ weighted). The shift from dominantly flow-regulated (predam) to grain-size–regulated (postdam) is the result of the reduced seasonal variability in discharge, resulting in a reduction in the denominator in (1) and (4).

β can be used to monitor bed sediment grain size, independently of changes in stage, by evaluating data from times when stage is within a narrow range. For example, a plot of changes in β through time, for discharges of 500–700 $m^3\ s^{-1}$, illustrates that the bed was relatively fine before the dam was built, coarsest during the 1980s (when postdam discharges were exceptionally high), and relatively coarse during the 1990s (Figure 9).

3.6. Example 6: Mississippi River

Suspended sediment concentration and median diameter were sampled in the Mississippi River at St. Louis between 1960 and 1972; several dozen measurements have been reported [*Scott and Stephens*, 1966; http://webserver.cr.usgs.gov/sediment/]. Analysis of these data suggests that although sediment transport is mainly flow-regulated, bed sediment changes are not negligible ($\alpha = -0.4$, both unweighted and weighted). As the stage increases, the supply of finer bank sediment that is available for transport on the bed increases (as was the case for the Paria and Colorado Rivers). These results are of interest because they show that grain size regulation of sediment transport may be measurable, even in an alluvial river.

4. Discussion

Distinguishing flow-regulated transport from grain-size–regulated transport is of utmost importance for sediment engineering problems, downstream from a dam, for example. Dams interrupt the supply of sediment to downstream reaches by trapping sediment. They also typically reduce peak annual discharge and increase minimum discharge. Where sediment transport is regulated mainly by flow, these two changes can have opposing effects on the downstream sediment budget: less sediment is supplied after a dam is built, but sediment supplied by tributaries downstream from the dam may be subject to reduced transport. Depending on the relative magnitudes of reduced supply and reduced transport, a dam may shift the sediment balance toward either net erosion, as reported by *Williams and Wolman* [1984], or toward deposition, as reported by *Andrews* [1986]. In some situations, however, the annual sediment load is determined not by water discharge but by tributary sediment input. In such a situation, changes in discharge caused by a dam might not change the sediment load; both predam and postdam sediment load may equal the tributary input.

To design dam releases for a particular engineering result, it is essential to know the relative importance of flow-regulated and grain-size–regulated transport. If changes in transport are regulated mainly by changes in flow, then reducing the peak discharge will shift the balance toward greater accumulation, as predicted by the sediment rating curve approach used in the Colorado River in Grand Canyon by *U.S. Department of the Interior* [1995]. In contrast, if sediment transport is regulated primarily by grain size of sediment on the bed, then sediment accumulation will be accompanied by fining of the bed, resulting in higher transport rates than would be predicted from rating curves that were developed for a constant grain size on the bed. To make predictions of sediment transport in such a system requires measurements of sediment input rather than just water discharge.

The Colorado River in Grand Canyon is a prime example of a river where the sediment budget is of critical importance. Instead of trying to predict the small difference between two large, poorly known quantities (sediment input and output), we examined the relation between size and concentration of suspended sediment in the predam and postdam rivers. Results indicate that predam transport at Grand Canyon was regulated mainly by flow but that grain size regulation was important during annual floods. In contrast, postdam transport is mainly grain-size–regulated.

A variety of other hydrologic characteristics can be suggestive of grain size regulation of sediment transport. For example, the rate of sediment discharge during floods is often greater during the rising limb than the falling limb because winnowing of the bed during the rising limb coarsens the bed, resulting in reduced transport (for comparable discharge) during the waning limb. This grain size evolution during a flood or annual water cycle causes rating curve hysteresis [*Leopold and Maddock*, 1953; *Meade et al.*, 1990; *Walling and Moorehead*, 1989; *Moog and Whiting*, 1998; *Topping et al.*, 2000]. Grain size evolution also causes upward coarsening of flood deposits [*Rubin et al.*, 1998]. Similarly, a rating curve with a large amount of scatter may hint at grain size regulation. A paucity of sediment stored in a channel may also be suggestive of grain-size–regulated transport; the lack of sediment suggests that the region may be deficient in sediment and also indicates that only limited supplies are available to replenish any sediment that is transported downstream during erosive flows.

Rivers are not the only environments where the balance between sediment supply (fining) and winnowing (coarsening) can vary through time. Such processes also have been reported from tidal flats [*Clifton and Phillips*, 1980; *Chun et al.*, 1998] and from field and modeling studies of transport on continental shelves [*Wiberg et al.*, 1994; *Drake*, 1999; *Harris and Wiberg*, 2000]. The results of this work should be easier to apply in these deeper marine settings, where the region of the bed that is accessible to the flow does not vary appreciably with flow strength.

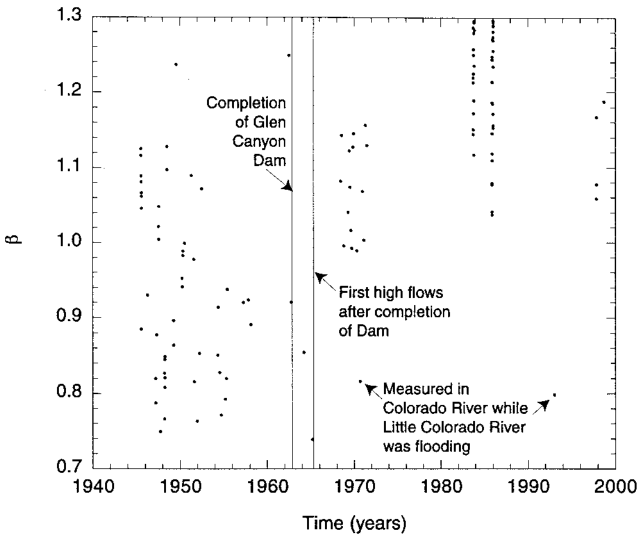

Figure 9. Plot of calculated relative coarseness of the bed (β) at the Grand Canyon gage for all flows with discharge between 500 and 700 $m^3\ s^{-1}$ (1944 to present). β was calculated using (11) and measurements of suspended sediment concentration and grain size. Bed was finest before the dam was built (1944–1962), coarsest in the 1980s (after exceptionally high discharges), and relatively coarse during the 1990s. Bed coarsened substantially during the first sustained high release from Glen Canyon Dam in April–June 1965, during which approximately 1.6×10^{13} grams of sediment were scoured from the reach between the Lees Ferry and Grand Canyon gages (using data presented by *Topping et al.* [2000]. The two subsequent low values of β in 1970 and 1993 are from measurements made at the Grand Canyon gage during large floods on the Little Colorado River, which temporarily increased the amount of finer sediment on the bed.

5. Conclusions

This paper presents several techniques for quantifying the extent to which evolving grain size of sediment on the bed regulates sediment transport in a time-varying flow α. The approach uses easily observable dependent variables C and D_s to calculate the independent variables D_b and u_* that determine sediment transport. The calculated changes in D_b and u_* are then evaluated for the degree to which each influences transport.

Results of such calculations can be used to guide sediment transport data collection and modeling efforts. In a flow where transport is regulated primarily by changes in grain size ($|\alpha| \gg 1$), measuring sediment input or grain size on the bed is more useful in predicting sediment transport than measuring flow. In contrast, where transport is dominantly flow-regulated ($|\alpha| \ll 1$), measurement efforts can focus on changes in flow, and models can assume a constant grain size on the bed. Where changes in both flow and bed sediment are important, data collection and modeling programs must account for both kinds of changes.

Sediment transport models can have at least three levels of sophistication in treating grain size of bed sediment. The simplest approach is to treat bed material as a constant. This approach is valid where transport is primarily flow-regulated; it requires the least field data (measurements of flow or discharge). The present paper treats evolving bed material as an independent variable or boundary condition rather than a constant. This approach has broader validity but requires more field data (flow or suspended sediment concentration and grain size on the bed or in suspension). A more elegant approach might be to treat bed material as a dependent variable that can be calculated from flow, sediment transport, initial bed grain sizes, and sediment mixing rates. This latter approach requires extremely detailed field data or model calculations: (1) spatial and temporal gradients in transport of each grain size fraction in transport, (2) the initial grain size distribution of sediment on the bed and in the substrate (where the bed undergoes erosion), (3) the thickness of the actively mixed sediment layer in the bed, and (4) the rate at which transported sediment is mixed with the substrate.

β can be used to quantify changes through time in grain size of sediment on the bed. β can change either because sediment on the bed evolves or because changes in stage give a flow access to different locations in a channel or within the substrate.

The techniques presented in the paper were developed theoretically and were tested with data collected in flume and field experiments. Application of the results indicates that Glen Canyon Dam changed the Colorado River in Grand Canyon from a flow-regulated to a grain size–regulated system. The dam also caused the bed to coarsen. Grain size regulation of transport can be important even in alluvial rivers such as the Mississippi.

Appendix A: Model Used to Predict Suspended Sediment Concentration and Grain Size

The suspended sediment concentration profile model used herein is based on *Smith and McLean* [1977] and *McLean* [1992] and is derived for steady, spatially uniform flow over dunes. In this model the effects of dunes were included because they were found to affect substantially the exponents in (2). The effects of density stratification (due to high gradients in suspended sediment concentration) were excluded because they affected the exponents in (2) in the text by no more than about 5%. All model calculations are performed in cgs units. Because the flow is approximated as steady and uniform, the total boundary shear stress is

$$\tau_b = -\rho g h S_f, \tag{A1}$$

where ρ is the density of water, g is the gravitational acceleration, h is the flow depth, and S is the water surface and bed slope. In steady, uniform, turbulent flow the vertical distribution of the total shear stress is linear [e.g., *Tennekes and Lumley*, 1972; *Middleton and Southard*, 1984] such that

$$\tau_{zx}(z) = \tau_b\left(1 - \frac{z}{h}\right), \tag{A2}$$

where $\tau_{zx}(z)$ is the total shear stress at each level z in the vertical.

For a given total boundary shear stress the presence of dunes on the bed reduces the shear stress in the lower portion of the flow [e.g., *Smith and McLean*, 1977; *Nelson et al*., 1993]. Thus, following the technique of *Smith and McLean* [1977] and *McLean* [1992], the total shear stress in the lower part of the flow is partitioned at each level z into a fluid component, i.e., a component related to the velocity and sediment transport, and into a form drag component because of the presence of the dunes. At the bed the fluid component of the stress is equal to the skin friction stress τ_{sf}, the stress available to entrain sediment from the bed into transport. The value of τ_{sf} is calculated by the *Wiberg and Smith* [1989] modification of the method of *Smith and McLean* [1977]:

$$\tau_{sf} = \tau_b \Big/ \left\{1 + \frac{C_D}{2}\frac{H}{\lambda}\left[\frac{1}{k}\left(\ln\frac{H}{(z_0)_{sf}} - 1\right)\right]^2\right\}, \tag{A3}$$

where $C_D = 0.2$ is the drag coefficient for separated flow over dunes [*Smith and McLean*, 1977; *Nelson et al*., 1993], H is the height of the dune, λ is the wavelength of the dune, $k = 0.408$ is von Karman's constant [*Long et al*., 1993], and $(z_0)_{sf}$ is the skin friction bed roughness parameter (determined by the method of *Wiberg and Rubin* [1989]). To solve (3), it is necessary first to specify values for the dune height and wavelength. On the basis of *Mohrig* [1994], and an analysis of the data of *Stein* [1965] and *Guy et al*. [1966],

$$\lambda = H(17 + 20X_s), \qquad X_s < 0.99 \tag{A4a}$$

$$\lambda = \infty, \qquad X_S \geq 0.99. \tag{A4b}$$

In (4), X_S is the suspended fraction of the bed sediment (i.e., the fraction of the bed sediment for which the skin friction shear velocity ($\sqrt{\tau_{sf}/\rho}$) is greater than the settling velocity). Also on the basis of *Mohrig* [1994] and an analysis of the data of *Stein* [1965] and *Guy et al*. [1966], $H \approx 0.3h$ when dunes are present on the bed.

For the detailed derivation of the suspended sediment concentration profile for each size class, see (6)–(14) by *McLean* [1992]. Because in this application of the model the clear water eddy viscosity of *Rattray and Mitsuda* [1974] was used and the effects of density stratification have been excluded, (14) of *McLean* [1992] simplifies to the following for each size class m of sediment at each level z:

$$\left(\frac{C_m}{1 - C_s}\right)_z = \left(\frac{C_m}{1 - C_s}\right)_a \left[\left(\frac{a}{z}\right)\left(\frac{h - z}{h - a}\right)\right]^p, \qquad z \leq 0.2h, \tag{A5a}$$

$$\left(\frac{C_m}{1 - C_s}\right)_z = \left(\frac{C_m}{1 - C_s}\right)_a \left[\left(\frac{a}{0.2h}\right)\left(\frac{0.8h}{h - a}\right)\right]^p \cdot \exp\left[-p\frac{B}{h}(z - 0.2h)\right], \qquad z > 0.2h. \tag{A5b}$$

In (1), C_m is the volumetric concentration of sediment in size class m, C_s is the total concentration of sediment in all size classes, a is the level at which the reference concentration is calculated (this level is different for the regions of the flow below and above one dune height above the mean bed elevation), $B = 6.25$ is a constant set by the matching height of $0.2h$, and p is the Rouse number,

$$p = \frac{w_m}{ku_*}, \tag{A6}$$

where w_m is the settling velocity of sediment in size class m (determined by the method of *Dietrich* [1982]) for sediment with a Powers index of 3.0, a Corey shape factor of 0.7, and a water temperature of 15°C, and u_* (the shear velocity) is set equal to $\sqrt{\tau_b/\rho}$ in the interior of the flow and $\sqrt{\tau_{sf}/\rho}$ below the tops of the dunes. The level a in the region within one dune height of the mean bed elevation is set equal to the elevation of the top of the bedload layer, as calculated by *Wiberg and*

Rubin [1989]. In the region above one dune height above the mean bed elevation, a is set equal to H, and C_m and C_s at a are determined by (5a). In the region of the flow below the tops of the dunes, C_m and C_s at a are determined by a linear form of the reference concentration of *Smith and McLean* [1977]:

$$(C_m)_a = i_m C_b \gamma \left(\frac{\tau_{sf} - \tau_{cr}}{\tau_{cr}} \right), \qquad \text{(A7)}$$

where $(C_m)_a$ is the near-bed time-averaged concentration of suspended sediment in size class m, i_m is the volumetric fraction of sediment size class m in the bed, $C_b = 0.65$ is the volumetric concentration of sediment in the bed, γ is a constant set equal to 0.0045 (P. Wiberg, personal communication, 2000) when a is determined by the method of *Wiberg and Rubin* [1989], and τ_{cr} is the critical shear stress of the median size of the bed calculated by the method of *Wiberg and Smith* [1987]. To preclude the occurrence of physically unrealistic high concentrations of suspended sediment in the cases where $(C_s)_a$ is predicted to be greater than 0.5 by (3), $(C_m)_a = 0.5 i_m$.

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