From Pathfinder to Glen Canyon: The Structural Analysis of Arched, Gravity Dams

by

David P. Billington
Chelsea Honigmann
Moira A. Treacy

Abstract

Shortly after its 1902 establishment the Reclamation Service embarked upon an ambitious program of designing and building large masonry dams in the west. The design engineers focused attention on the problem of high dams curved in plan such that the behavior was partly as a horizontal arch and partly as a vertical cantilever. This physical reality for dam sites in the first decade of the 20th century posed a challenge to the analytic talents of engineers and called forth an approach that eventually in the 1920s came to be called the trial-load analysis.

In March 1903 the Reclamation Service hired George Y. Wisner as their structural consultant and in 1904, with E. T. Wheeler, he embarked on a mathematical study focused on the Pathfinder Dam. Their report, published in 1905, identified the two types of behavior – horizontal arch and vertical cantilevers – and showed how a highly approximate approach could be used to estimate the overall performance of curved masonry or concrete dams. The Service used the result for the design of Pathfinder and used it to check the design for the Theodore Roosevelt Dam.

In a 1921 landmark paper, Fred Noetzli, a Swiss trained engineer, developed a more complete procedure for the arch-gravity dam analysis which he applied to Pathfinder and found results reassuringly similar to those published by Wisner and Wheeler.

After much published discussion of Noetzli's paper, C. H. Howell and A. C. Jasquith presented, in a 1929 paper, a more extensive procedure and for the first time used the term trial load as the method of defining the extent of the arch action and that of the cantilever action. Hoover dam, then under design, did not benefit from this analysis but in the 1950s the trial-load method helped justify the design for Glen Canyon Dam, which unlike Hoover Dam, could not stand safely as a pure gravity structure.

The paper will conclude with some general observations about the role of conceptual design, based on approximate methods of analysis, in the search for structural forms that are sufficiently safe and relatively economical. Also included will be a related discussion about the tension between the massive and structural traditions of concrete design.

From Pathfinder to Glen canyon: The Structural Analysis of Archied, Gravity Dams

by

David P. Billington[*]
Chelsea Honigmann[+]
Moira A. Treacy[x]

The Beginning of Rational Design

Structural engineering as a modern profession begins with the building of iron bridges in the late 18th century in Great Britain. It began because of the desire for lighter bridges that could nevertheless be as strong or even much stronger than those built of stone or wood. Starting with the French schools, the Ponts et Chaussées established in 1748 and the Ecole Polytechnique established in 1794, structural engineering by the early 19th century began to have a foundation of a scientific basis where mathematical theory could help predict performance and be therefore a guide to designing new forms.

Bridges were the primary focus of early structural theory because they were pure structure, they had the longest spans, and they also had the most dramatic failures. During the last half of the 19th century structural theory became formalized, began to be used extensively for buildings, and was taught systemically in the Polytechnique Institutes of Western Europe. By contrast with bridges and buildings, dams did not receive the same intensive attention in schools or in the technical literature. This was so because most dams were low, were built of earth or rock, and thus remained part of a preindustrial technological culture. Throughout the 19th century dams received little attention either in the technical literature or in schools of engineering. But at the

[*] Professor of Civil and Environmental Engineering, Princeton University
[+] Graduate Student, Dept. of Civil and Environmental Engineering, Princeton University
[x] Former Graduate Student, Dept. of Civil and Environmental Engineering, Princeton University

end of the century three major changes in the United States brought dams into the forefront of engineering: first, cities were expanding at an unprecedented rate and they could not grow without new sources of water; second, the new electric power industry moved rapidly into hydroelectric stations; and third, the closing of the frontier raised strong social pressure to develop the west in large part through irrigation.

Those social pressures combined with the advanced state of structural theory and produced the desire for a more scientific treatment of dams with the belief that they could therefore be more rationally, hence more economically and more safely, designed. Just at this time the new and prototypical 20th century material, structural concrete, came into general practice to encourage designers to abandon stone masonry and sometimes embankment dams for ones built using the new material. But even where earth or rock dams seemed still preferable, concrete became widely used in spillways, powerhouses, and diversion works.

In addition to these social and technical forces there was the crisis of flood to prompt federal funds for control dams. The political actions that such floods bring naturally result in population growth and urban expansion. As the 20th century unfolded, the major dam building in the United States and elsewhere began to take a new direction, characterized by high dams, huge reservoirs, and the search for rational methods of analysis as a basis for design. This search led to two competing visions of structural form, one characterized by the structural tradition and the other by the massive tradition which we can rephrase as the battle between form and mass.[1]

Form and Mass in Structure

In the preindustrial world, with the notable exception of the high gothic cathedrals, there was an implicit belief that great works were built as massive structures which were primarily of stone. This aesthetic of mass connoted permanence, opulence, and power; it stood in opposition to the ephemeral wooden structures of peasants and the urban poor. To be monumental was to be safe and handsome. When engineers began to construct skeletal metal bridges in the 19th century, they were initially banned from urban settings and when concrete entered practice in the 1890s it had to be covered in, or formed to look like, stone to be accepted.

It is therefore of no surprise that when large dams entered modern America of the 20th century, they would reflect that context, especially those dams designed by large municipalities and agencies of the Federal Government. And yet right from the start of federal dam building in concrete, with the founding of the Reclamation Service in 1902, the conflict between form and mass was immediately present and it would remain as a continuing issue, never fully resolved, throughout the century.

In its most elementary form, a dam in the massive tradition consists of a mass of material that - by its weight alone - holds back a volume of water. Such structures are known as gravity dams, an appropriate name because it is the force of gravity pulling vertically down on the dam that provides resistance against pressure exerted horizontally by water in the reservoir. Designs adhering to the massive tradition can be based upon sophisticated engineering analysis, but the basic principle underlying the tradition is simple: accumulate as much material as economically or physically possible, thus insuring that the dam will not tip over, slide or rupture; in turn, the massive dimensions will increase the likelihood that the dam can achieve long-term stability in holding back a reservoir.

A dam in the structural tradition, in contrast to gravity designs, depends upon its shape - and not simply its mass - to resist hydrostatic pressure. For example, an arch dam in a narrow canyon with hard rock sides allows a significant amount of the hydrostatic pressure to be carried by arch action horizontally into the canyon walls. Because of this arch action, the thickness (and hence bulk) of the dam’s profile can be much less than a gravity dam of the same height. In essence, the amount of material in (or the mass of) a structural dam is a less important attribute than it is for a massive dam. For a dam adhering to the structural tradition it is more important to develop a design that takes advantage of shape and not just weight.

The Profile of Equal Resistance

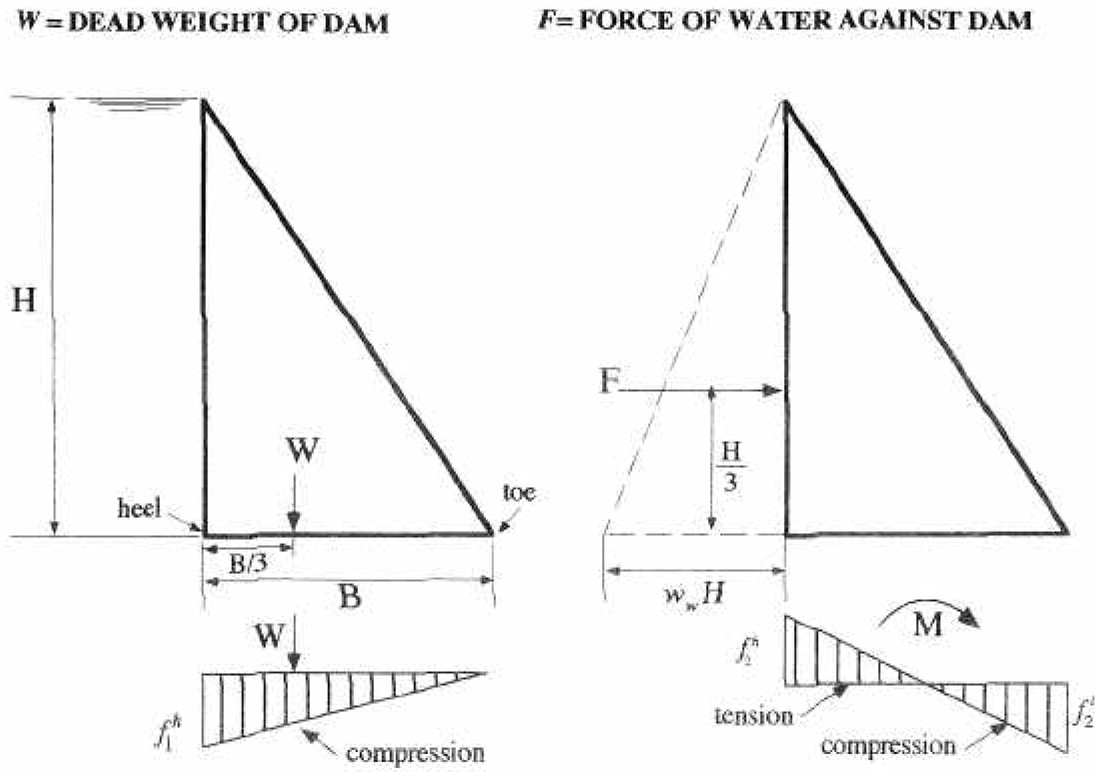
Masonry gravity dams can be built without any reliance upon mathematics, but in the 19th century European engineers realized that this type of structure was amenable to a quantifiable approach to design. In the early 1850s a paper published by the French engineer J. Augustine DeSazilly set the course for all subsequent work in this area of gravity dam design.[2] Knowing the hydrostatic force exerted by a given height of water (which weighs about 62.5 pounds per cubic foot) and the approximate weight of masonry used in dam construction (usually about 140-150 pounds per cubic foot), DeSazilly conceived what he termed the "profile of equal resistance". Using basic formulas of statics, he developed a cross-section in which compressive stresses at the upstream face when the reservoir is empty equal compressive stresses at the downstream face when the reservoir is filled. In taking these two extreme conditions, he hypothesized a design that, at least in cross-section, would minimize the material necessary to erect a stable masonry gravity dam.

The profile of equal resistance came from a consideration of two major conditions of dam loading (see Figure 1): reservoir empty or reservoir filled. For the former case the dead load of the dam, assumed to be a pure triangle in cross section, caused a maximum vertical compressive stress f_1^h at the heel of the dam (upstream edge) equal to the weight of concrete or stone above that point or $f_1^h = Hw_c$ (Height H times the density of concrete w_c). For the case of the full reservoir, to the vertical stress of case one must add the influence of the horizontal force F due to water pressure. This force causes the dam to bend and thus creates maximum vertical compressive stresses at the toe $f_2^{\mathrm{t}} = Hw_w\left(\frac{H^2}{B^2}\right)$ with equal vertical tensile stress at the heel (see Figure 1). The criterion for equal resistance is that the maximum vertical compressive stress for case one be the same as for case two, hence $Hw_c = Hw_w\left(\frac{H^2}{B^2}\right)$ or $\frac{H^2}{B^2} = \frac{w_c}{w_w}$. For example where the density of concrete w_c is taken to be 140 pounds per cubic foot and the density of water w_w to be 62.5 pounds per cubic foot then $\frac{H^2}{B^2} = \frac{140}{62.5} = 2.25$ so that $\frac{H}{B} = \sqrt{2.25} = 1.5$ or about 3/2. For example, for a dam 60 feet high the base width would be 40 feet.

The Middle Third

In the early 1870s, the Scot W. J. M. Rankine confirmed the validity of DeSazilly's work; he further observed that a stable gravity dam must have sufficient cross-section so that the combined vector force (or "resultant force") of the horizontal hydrostatic pressure and the vertical weight of masonry will pass through the center (or middle) third of the structure at any horizontal elevation.[3] Should the resultant fall outside the center third, a gravity dam will become susceptible to dangerous cracking because tension (rather than compression) will develop along the upstream edge of the structure; the further outside the center third the resultant passes, the greater the tensile stress and the greater the likelihood that cracking will occur. And if the resultant should fall completely beyond the downstream edge, then the structure will "overturn." Although the "middle third" precept was inherently adhered to by any design developed in accord with by De Sazilly profiles, Rankine's work established it as an overt principle of masonry gravity design.[4]

When the stresses for case two are plotted over the dam base we find that they form a triangle with the maximum value at the toe and the minimum (equals zero) at the heel. The centroid of that pressure lies at B/3 from the toe. Likewise for the reservoir empty in case one



$$W = \frac{HBw_c}{2} = \frac{f_1 B}{2}$$

$$f_1^h = Hw_c$$

$$M = \frac{FH}{3}; \quad F = \frac{(w_w H)H}{2}$$

$$M = \frac{H^3 w_w}{6}$$

$$f_2 = \frac{M}{S}; \quad S = \frac{B^2}{6}$$

$$f_2^t = \frac{H^3 w_w}{B^2} = -f_2^h$$

For equal resistance, $f_1^h = f_2^t$

$$Hw_c = \frac{H^3 w_w}{B^2}; \quad \frac{H^2}{B^2} = \frac{w_c}{w_w}$$

Where $w_c = 140$ lbs. per cubic foot and $w_w = 62.5$ lbs. per cubic foot

$$\frac{H^2}{B^2} = \frac{140}{62.5} = 2.25 \text{ and } \frac{H}{B} = \sqrt{2.25} = 1.5 = \frac{3}{2}$$

Figure 1 - Profile of equal resistance

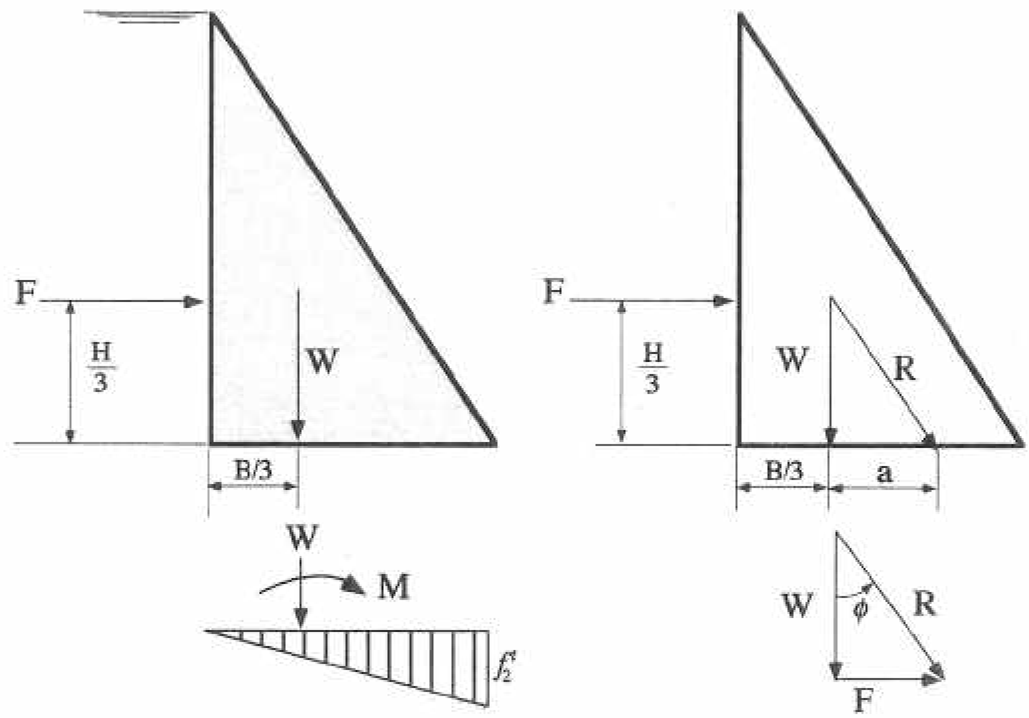
the centroid lies at B/3 from the heel. Thus, the centroids of all loading cases between one and two lie between those two positions or within the middle third of the dam width B (see Figure 2).

The Danger of Uplift and Sliding

Interest in other issues relating to gravity design did not remain stagnant and this is best reflected in concern over the influence of uplift on the safety of gravity structures. Uplift is a phenomena resulting where water seeps under the foundation (or into the interior of the dam proper) and — because of pressure exerted by water in the reservoir — pushes upward and increases the likelihood that the structure will slide horizontally downstream. Uplift attracted the attention of engineers in the early 20th century and encouraged both the use of thicker profiles as well as the development of grouting and drainage techniques that would mitigate its occurrence and possible effect.

The 1911 failure of a gravity dam in Austin, PA led the American engineering profession to look more closely at the influence of uplift on dam safety especially as it related to sliding. Figure 3 illustrates the forces that influence the horizontal movement of a gravity dam over its foundation. In addition to the force of the water F and weight of the dam W, the water pressure underneath the dam produces uplift U while the cohesion C between dam and rock resists sliding. The friction between dam and foundation (usually rock) tan ϕ will resist sliding in proportion to the vertical force W less the uplift. Neglecting cohesion and assuming full uplift on a dam where B/H = 2/3, the safety factor against sliding is less than one. This result helps explain the Austin Dam failure, where B/H = 0.6 and investigations after failure led to the conclusion of substantial uplift. Part of the solution was to increase B/H and also to drain the base to relieve the pressure (see Figure 3) and hence reduce the uplift force to 0.5 or less.[5]

The most significant drawback to gravity designs involved their high cost. While the “profile of equal resistance” offered a mathematically rational basis of design, this did not mean



$$F = \frac{HBw_c}{2}$$

$$F = \frac{H^2 w_w}{2}$$

$$Tan\phi = \frac{F}{W} = \frac{a}{H/3}$$

$$= \frac{H^2 w_w / 2}{HBw_c / 2} = \frac{H}{B}\frac{w_w}{w_c}$$

$$\frac{3a}{H} = \frac{H}{B}\frac{w_w}{w_c} \quad \text{or} \quad a = \frac{H^2}{3B}\frac{w_w}{w_c}$$

For $w_w = 62.5$ lbs. per cubic foot and $w_c = 140$ lbs. per cubic foot

$$a = \frac{H^2}{B^2}\left(\frac{1}{2.25}\right)\frac{B}{3} \quad \text{so that when} \quad \frac{H}{B} = \frac{3}{2}$$

$$a = 2.25\left(\frac{1}{2.25}\right)\frac{B}{3} = \frac{B}{3}$$

Thus, a defines the middle third of the base. If the resultant R falls outside a, then there will be tension at the heel and the compression at the toe, $f_1^t + f_2^t$ will exceed f_2^h.

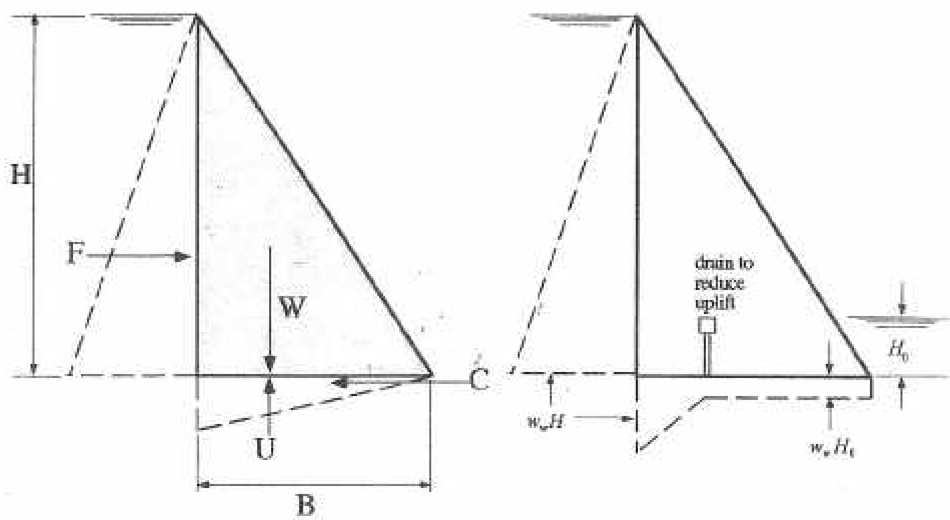
Figure 2 - The middle third

that gravity dams would necessarily be cheap to build. For major municipalities, the economic benefits that accompanied an increased water supply might easily justify the huge expenditures required to build large masonry gravity designs. But once cities such as Boston (with the Wachusetts Dam completed in 1904) and New York (with the New Croton Dam completed in 1907) erected masonry gravity structures as part of major civic improvement projects, the technology came to represent — at least in many people's eyes — the most conservative, the most appropriate, and, if at all economically feasible, the most desirable type of dam. In such dams, the free end (top) of a straight gravity dam will move horizontally as the cantilever bends downstream under water pressure. In this way the water load is carried down to the foundations (on the valley floor) by bending.

Arch and Cantilever Behavior in Dams

Unlike the Croton structure, many dams in narrow valleys have been designed as arches using an elementary mathematical theory based upon the cylinder formula (see Figure 4). The dam, curved into an arch between the sides of the valley, will carry water load to the vertical canyon walls, by compression forces calculated from the cylinder formula. As these horizontal arches carry compression they will become shorter and hence move in the horizontal direction downstream. Thus a curved arch dam can carry loads both vertically as a cantilever and horizontally as an arch. The challenge to the engineer is to determine how much of the load goes to the canyon floor and how much to the canyon walls.

This issue is crucial to design because much more material is required for safe cantilever behavior than for safe arch action. For example, designers proportioned gravity dams (those assumed to act as cantilevers alone) with a base thickness B equal to about 2/3 of H, the dam height. Where H = 60 ft. and B = 40 ft. the amount of concrete required per foot of dam length would be V = 60 x 40 x 1/2 = 1200 cubic feet. By contrast an arch dam with H = 60 ft. would



$F = \dfrac{w_w H^2}{2} =$ Force of water

$W = w_c \dfrac{BH}{2} =$ Weight of concrete in dam

$U = m w_w \dfrac{BH}{2} =$ Uplift due to water pressure

$C = cB =$ Cohesion between concrete and bedrock

$Tan\phi =$ Coefficient of friction between dam and bedrock

Sliding Force $F - C = (W - U)\,Tan\phi$

Sliding will occur if $F - C > (W - U)\,Tan\phi$

or when $\dfrac{F - C}{W - U} > Tan\phi$

The safety factor $SF = \dfrac{W - U}{F - C} Tan\phi$

For an indented bedrock $Tan\phi$ can be one so that

for full uplift $m = 1$, $\dfrac{B}{H} = \dfrac{2}{3}$ and $c = 0$,

$$SF = \frac{BH}{2}\left(\frac{w_c - w_w}{w_w H^2 / 2}\right) = \frac{B}{H}\left(\frac{w_c}{w_w} - 1\right) = \frac{3}{2}(1.24) = 0.83\,*$$

Where a drain reduces uplift $m = 0.5$,

$$SF = \frac{3}{2}(1.74) = 1.16$$

Where the base width equals the height and m = 0.5 ,

$$SF = 1.0\,(1.74) = 1.74$$

*When SF (safety factor) is less than one, the dam fails.

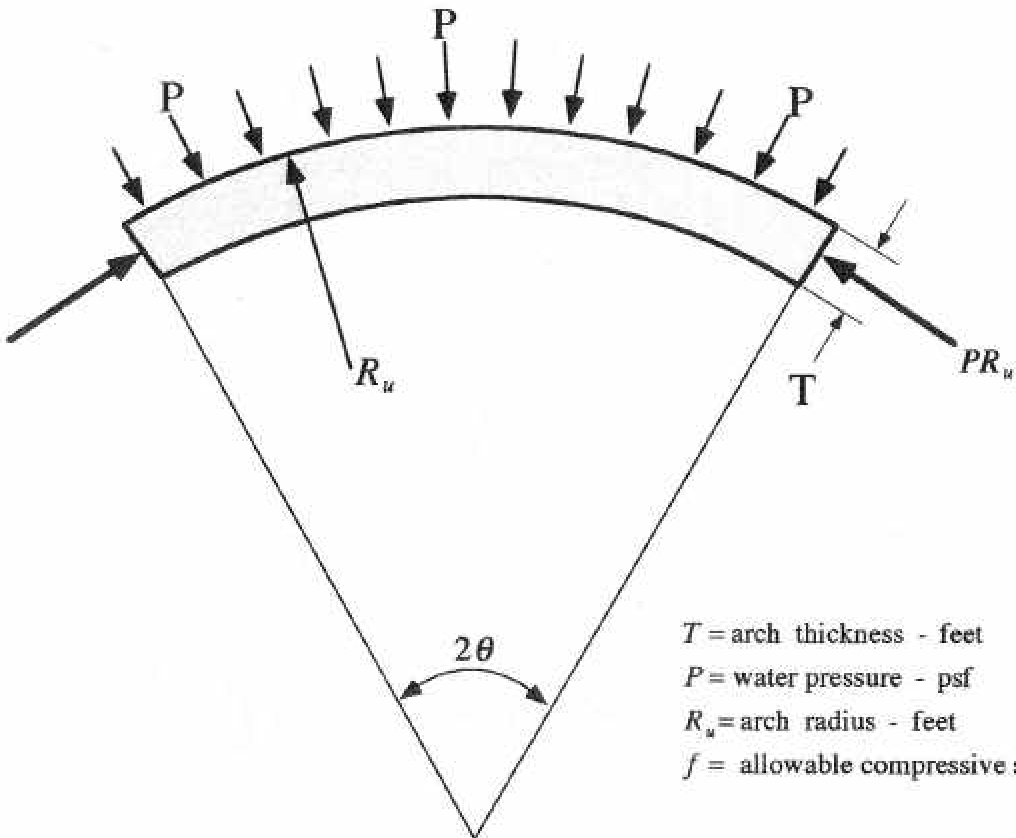
Figure 3 - Uplift and Sliding

require a base thickness of about 7.5 feet from the cylinder formula (for f = 350 psi) and hence a total volume of 60 x 7.5/2 = 225 cubic feet or less than 20% of the material required for the gravity or massive dam.

As a result, some engineers, seeing this great advantage of arch dams, had a strong incentive to find a rational way to determine analytically how much load was carried by the arching action and thereby justify designing a safe dam with far less material than a gravity dam carrying load by cantilever action. Engineers consulting with the newly established Reclamation Service began this process of analysis as early as 1903.

The Wisner and Wheeler Report on Pathfinder Dam

In September 1903, the Reclamation Service held a conference of engineers at Ogden, Utah, where their newly appointed (March, 1903) consulting engineer, George Y. Wisner (1841-1906), presented a paper which called for a thorough study of stresses in high masonry (stone or concrete) dams to ensure safety and achieve minimum construction cost.[6] F. H. Newell, the chief engineer of the Service, asked a select committee of four, including Arthur Powell Davis (1861- 1933) later to become director of the Service, to make him a recommendation which it did formally on October 5, 1904. Its letter spoke of the two high dams proposed for Wyoming (Pathfinder and Buffalo Bill) and of the fact that "no thorough analysis has ever been made of the relative economy and stability of reinforced concrete dams as compared with similar dams of gravity sections..." They suggested that such an analysis be commissioned by this Service and they recommended Mr. E. T. Wheeler of Los Angeles for the job.[7] Under the supervision of Wisner, Wheeler began work in January of 1905. Wheeler submitted his final report on May 5, 1905 and Wisner sent that report, preceded by a lengthy discussion of his own, to Newell on May 16. Its importance was considered to be so great that the Wisner-Wheeler paper was published in the August 10, 1905 issue of *Engineering News*. Since this report



T = arch thickness - feet
P = water pressure - psf
R_u = arch radius - feet
f = allowable compressive stress - psf

$$T = \frac{PR_u}{f}$$

The cylinder formula derived by the French mathematician Louis Navier in 1826 postulates that the thickness of cylinders $\boldsymbol{T}$ placed under hydrostatic pressure $\boldsymbol{P}$ and having a compressive stress $\boldsymbol{f}$ is calculated according to the equation:

$$P = Hw_w$$

$$T = \frac{PR_u}{f} = \frac{Hw_wR_u}{f}$$

thus, the thickness varies directly with $\boldsymbol{H}$, if $\boldsymbol{R_u}$ and $\boldsymbol{f}$ remain constant

Figure 4 - The cylinder formula

inaugurated the structural tradition of large-scale dam design within the Federal Government of the United States, it is essential to explain its substance and its impact.[8]

Although Wisner proposed the study in the light of the Service's new big dams — Roosevelt, Buffalo Bill, and Pathfinder — he and Wheeler actually focused only on Pathfinder (see Figure 5), it being the first one to be completed (1909). Wisner described how an arch dam in a narrow valley (he called it "of short span") carried water loads and also how it behaved under wide swings of temperature both with reservoir full and with it drawn down. He then gave

Figure 5 - Pathfinder Dam on the North Platt River near Casper, Wyoming is a masonry arch and cantilever section.
Source: Bureau of Reclamation

Wheeler's report which consisted of the sets of formulas for water loads: one which assumed that the dam carried the water pressure as a series of horizontal arches supported by the side walls of the Canyon. He then computed the horizontal deflection of these arches at their crowns — essentially only the vertical centerline of the dam (see Figure 6).

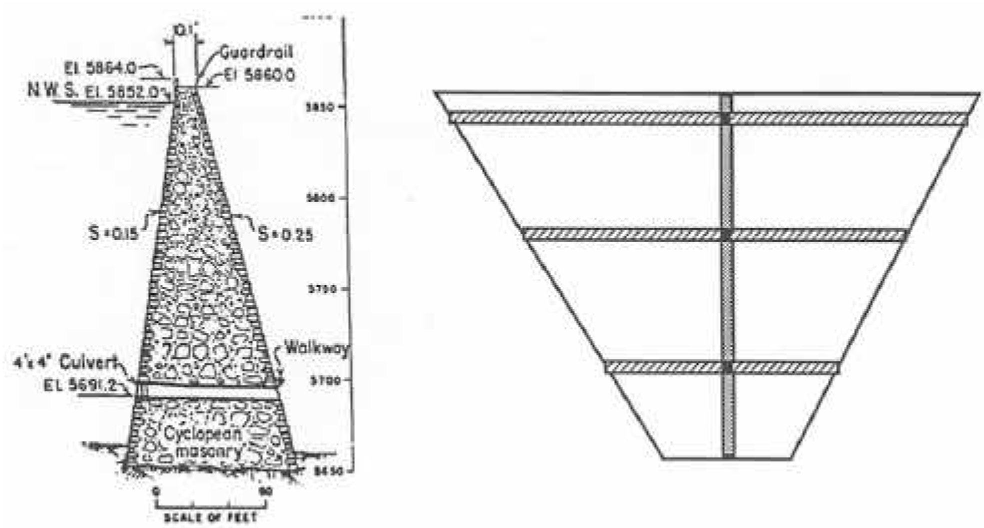


Figure 6 - Pathfinder Dam section and elevation diagrams. Wheeler took a vertical slice of the dam as a cantilever and analyzed the deflections. He then analyzed the deflections in a horizontal arch section. He repeated these analysis, distributing load to both the arches and cantilever, until the two sets of deflections were nearly equal.

Wheeler next took a vertical slice of the dam at this centerline and, assuming it carried all the water pressure as a cantilever, supported only on the floor of the Canyon, he computed its horizontal deflection at various points from base to top of the dam. The arch deflections and the cantilever deflections should have been the same at the same points on the dam, but this two-part calculation did not give such results. Thus Wheeler had to make a second calculation by adjusting the amount of load taken by the arches and that taken by the cantilevers. The first calculation shows that the free cantilever deflects far more than the arches do in the top portion of the dam while the reverse is true at the bottom. Thus the arches carry more load at the top and the cantilevers carry more load at the base. This redistribution of load would eventually be called the trial-load method of analysis. Moreover, Wheeler found that the Pathfinder dam could carry all the water load as a series of arches with compressive stresses under 200 psi for a material (stone masonry) whose compressive strength is well over 2000 psi.

Next Wheeler studied temperature stresses in the Pathfinder dam. Here he assumed that the temperature dropped 15°F at the top with the reservoir only filled up to 100 ft. from the top and that the temperature drop decreased linearly to zero at 120 ft. below the top. This drop would cause the arches to bend and deflect in the downstream direction that would cause vertical cracks in the upper arches; and the deflection of the arches above relative to the undeflected cantilevers below would cause vertical bending in the lower parts of the dam and hence horizontal cracks there. This qualitative description helps explain where reinforcing steel needs to be placed (if it were a concrete dam) but it does not give a good quantitative measure. However, by iteration again Wheeler was able to make a more reasonable estimate of the temperature stresses which he then combined with the water load to give one design condition.

Noetzli and the Curved Dams

Strictly speaking, the analysis of Wisner and Wheeler was a trial-load method because it assumed a distribution of loads between arches and cantilevers and then after various other trials it based design on a final iteration. Fred Noetzli (1887-1933), a Swiss trained engineer, summarized the situation in a landmark 1921 paper in which he reviewed the practice of arched dams, gave relatively simple formulas for calculating the cantilever and the arch actions in horizontally curved dams, and then applied his formulations in detail to the Pathfinder Dam. This last part is the heart of his paper in which he compares his semi-graphical approach to the purely analytical calculations presented by Wisner and Wheeler in 1905. He concludes that his "distribution of load between cantilever and arches compares very favorably with that obtained analytically by Mr. Wheeler."[9]

Noetzli then proceeded to discuss the central issues in dam design that went beyond the statics of water-pressure loading: stresses due to temperature change, to shortening of the arches under water pressure, and to shrinkage of the concrete as well as the influence of cracks in the concrete. He showed by simple calculations that these effects were at least as important as those due to the statics of water pressure loading.

The benefit of analyzing the dam as a set of independent arch and cantilever elements is that one can use simple calculations to determine the deflection of any point along an arch or cantilever element. Cantilever deflections are approximated using the moment area method. The height at which a deflection will occur along the cantilever can be calculated for members with constant and linearly varying cross-sections using simple equations. These equations are given below:

The arch deflections are analyzed assuming pinned ends because of the relative flexibility

$$H = \sqrt[5]{\frac{yEt_b^3}{p}} \quad \text{(triangular cross-section)}$$

$$H = \sqrt[5]{\frac{2.5yEt_b^3}{p}} \quad \text{(constant cross-section)}$$

caused by the overall arch length.

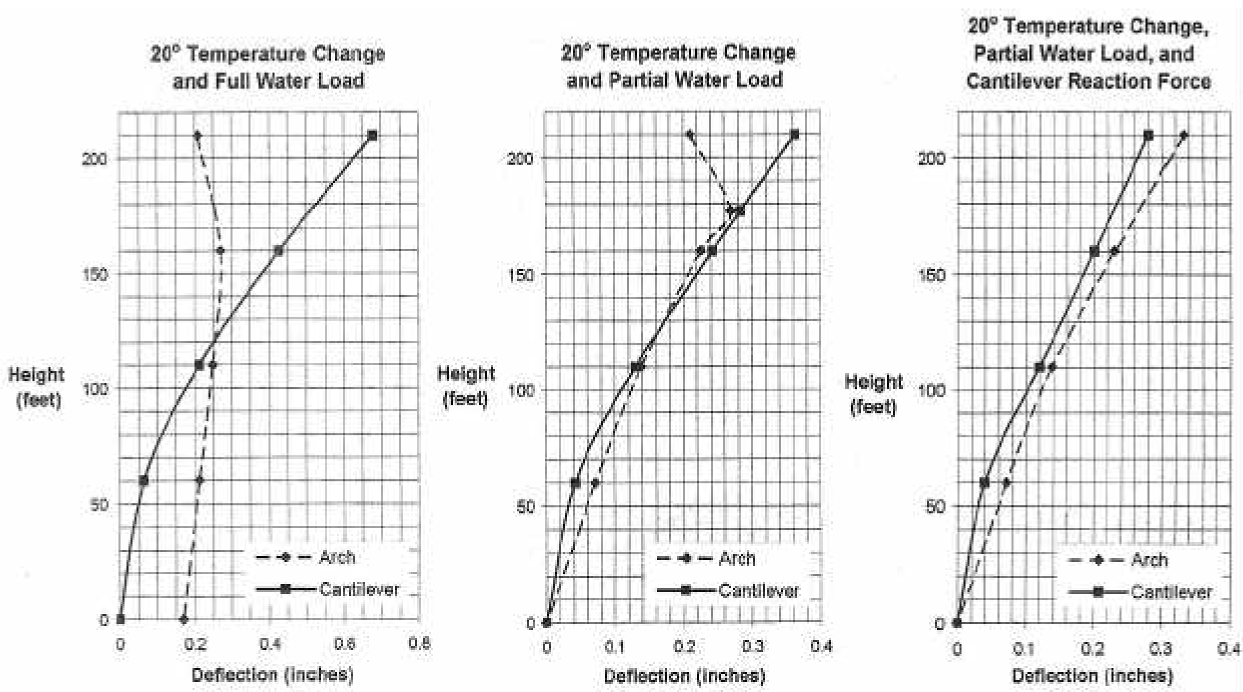
The process of determining the load distribution between arches and cantilevers in a given

$$\Delta_{arch} = \frac{1.56}{E} \cdot \frac{PRR_u}{t}$$

dam is an iterative approach based on finding the height, H', above which cantilever action no longer exists and arch action takes all the load. A first approximation of H' is made by applying the full water load individually to the arch and cantilever elements. The arch and cantilever deflections will coincide at a single point. The results of an independent analysis of the Pathfinder

Dam based on the method outlined by Noetzli in his 1921 paper are shown in Figure 7 below. Figure 7a shows the intersection of the arch and cantilever elements at a deflection of approximately 0.25in. This value is used in the two cantilever equations above to calculate the height at which this deflection would occur in an idealized structure. As the actual cross-section of the dam is something between prismatic and triangular, the first approximation of H' is taken as the average of the results from the two equations.

The load distribution defined by this new value of H', becomes the basis for the second



a) b) c)

Figure 7 - Deflection of Pathfinder Dam, based on Noetzli's analysis. Deflection of arches and crown cantilever under combined 20° temperature change and a) full water load, b) under partial water load, and c) under partial water load and reaction from cantilever.

iteration. The deflection of the arch and cantilever elements is calculated as before for the new loading and is shown in Figure 7b.

Although the cantilever carries no load above H', the deflection continues to increase up to the waterline. In some cases, the calculated cantilever deflection may exceed the arch deflection at the top of the dam, as it does in Figure 7b for the Pathfinder Dam. In actuality, however, the deflection of the cantilever and arch must coincide, requiring that the arch must resist and additional deflection of the cantilever. Subsequent iterations involve adjusting the value of H' and finding, by trial and error, the additional "reaction" load on the arches required to bring the cantilever deflection approximately into coincidence with the arch deflection, as shown in Figure 7c.

Table 1 summarizes Noetzli's results for the Pathfinder Dam and the results of the independent check of Noetzli's analysis. Additionally, a finite element model was analyzed using SAP 2000® to check the accuracy of Noetzli's values. These results are also given in Table 1. In general, the finite element results compare favorable with those obtained by the simplified hand analysis, indicating the applicability of this method for approximating the behavior of arch dams

Table 1 - Comparison of Pathfinder Dam deflections from several analysis methods

	Noetzli's Results	Independent Check of Noetzli's Trial Load Method			SAP 2000 Analysis
		Arch Defl'n			
Elevation	Total (inches)	D_{temp} (inches)	D_{water} (inches)	Total (inches)	Total (inches)
210	**0.320**	0.211	0.121	**0.33**	**0.23**
160	**0.230**	0.154	0.066	**0.23**	**0.15**
110	**0.140**	0.11	0.028	**0.14**	**0.08**
60	**0.070**	0.06	0.012	**0.07**	**0.04**
0	**0.000**	0.000	0.000	**0.00**	**0.00**

under the combined effects of temperature and water load.

After giving the details of his analysis of the Pathfinder Dam, Noetzli went on to point out that pure gravity dams rarely have a safety factor against overturning of over 2.0 and usually it is close to 1.0 (see Figure 8). This surprising claim allowed him to make a strong criticism of such dams, i.e. “no other engineering structure of acknowledged good design has such a small factor of safety as a pure gravity dam.”[10]

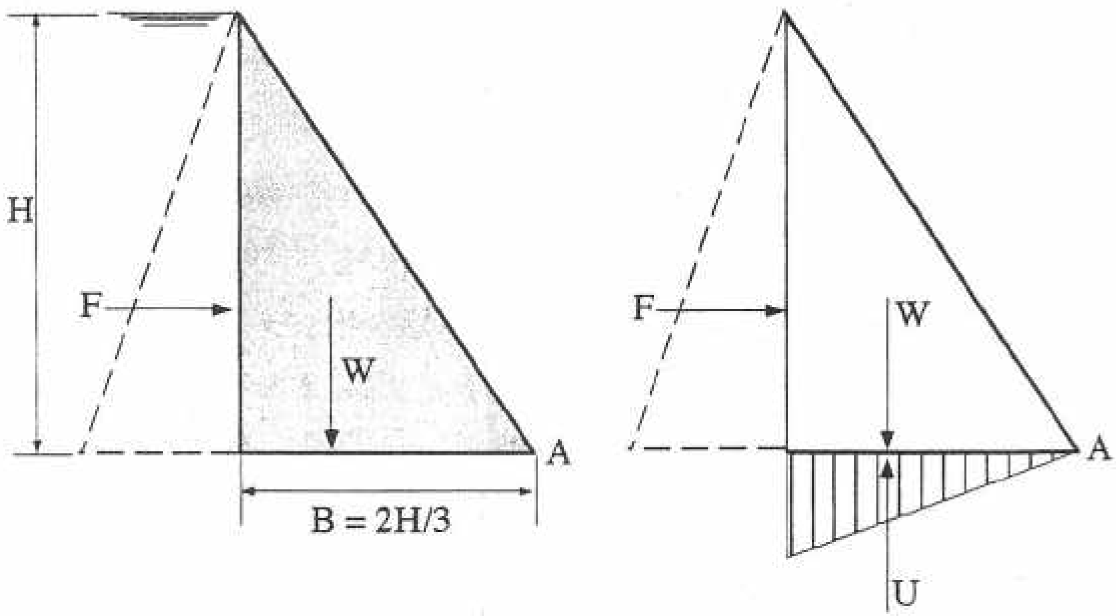
The paper, which drew vigorous discussions from major figures of the period, established the Swiss engineer as a leading theoretician for dams and the discussion, largely confirmed Noetzli’s reputation. Running through Noetzli’s writing was the two-part theme, prototypically Swiss, that good design implies form over mass and that analysis—often graphically done—can be greatly simplified to improve understanding as well as to encourage designers to think in terms of form over mass. He was at great pains to stress the historical fact that mass did not mean safety but that form, properly conceived, did so and with greater economy as well.

Much of the discussion revolved about the relative simplicity of the graphical approach as compared to the complexity of the mathematical one.[11] One factor in the form versus mass debate was the perception that lighter forms needed more rigor in solution.

The Trial-Load Method

The writings on curved dams continued throughout the 1920s as the nation was beginning to move into the largest program of dam building ever attempted. The articles and discussions up to 1929 discussed both arch and cantilever behavior and hence qualify as trial-load methods. However, not until the 1929 ASCE *Transactions* by C. H. Howell and A. C. Jaquith does the method acquire publicly the name of “trial load.”[12] Both authors had worked for the Bureau of Reclamation in Denver where they had begun to study the method in 1923.

In the paper they defined the method as one which considers the dam to be made up of a series of horizontal arches and a series of vertical cantilevers with part of the water load carried by the dam considered as arches and part by the dam considered as cantilevers. The arch loads



Overturning about point A (toe)

$$SF = \frac{W2\left(\frac{B}{3}\right)}{F\left(\frac{H}{3}\right)} = \frac{w_c B^2\left(\frac{H}{3}\right)}{w_w\left(\frac{H^3}{6}\right)}$$

$$= \frac{w_c}{w_w} 2\frac{B^2}{H^2}$$

$$= 2.25\ (2)\left(\frac{4}{9}\right) = 2.0$$

$$SF = \frac{(W-U)\ 2\left(\frac{B}{3}\right)}{F\left(\frac{H}{3}\right)} = \left(\frac{w_c}{w_w} - 1\right)\ 2\left(\frac{B^2}{H^2}\right)$$

$$= 1.25\,(2)\left(\frac{4}{9}\right) = 1.1$$

Figure 8 - Factor of safety against overturning

and the cantilever loads are adjusted so that the deflection of the arches are nearly the same as the deflections of the cantilevers at the same points. They distinguish the trial-load method from previous similar methods by the fact that they were considering more than the one single cantilever, which is what Wheeler, Noetzli, and others had done. By considering a series of cantilevers, rather than one cantilever only at the centerline of the dam, the Bureau engineers created a detailed procedure which was used for later dams. In their paper, the authors began by noting the variations in the shapes of canyons in which dams appear and thus they established the need to use more than one cantilever for more realistic analysis.

As with Noetzli's paper, the Howell and Jaquith paper brought forward much substantial discussion. Noetzli and Jakobsen both observed that Alfred Stucky had used the trial-load method for a Swiss dam in 1922 although the method was not so named. In fact Robert Maillart had used the same idea in 1902 for a water tank also in Switzerland.[13] Probably the most significant discussion from the point of view of federal dams came from John Savage and Ivan Houk, both of the Bureau. They gave a more detailed discussion of Gibson Dam and gave also results from their analysis of the 405 ft. high Owyhee Dam in eastern Oregon. Savage had assumed a dominant role in the Bureau and was already in 1929 deeply involved with the Boulder Canyon project. But as the dams got higher and higher, the Bureau recognized the need to develop not just mathematical analyses but also physical model testing, and the instrumentation of full scale dams.

The Stevenson Creek Test Dam

During the first three decades of this century engineers focused intently on concrete arches, creating numerous designs for bridges as well as dams, and stimulating more mathematically complex analytic schemes. In 1924, four of the twenty *Transactions* papers dealt with concrete arches and many of these pages were filled with formulas and tables. The 1925 *Transactions* contained two extensive articles on arch analysis, in total about 20% of the entire volume.

But already by 1922, some engineers became uneasy with so much abstraction and began to worry about field performance as opposed to office abstractions. Particularly engineers in the western states saw the need for a different approach to analysis which led Fred Noetzli to request financial support from the Engineering Foundation for collecting performance data on existing arch dams and for designing new tests and experiments.[14] Noetzli noted the national significance of arch dams by referring to two recent papers on the subject which had won the Croes Medal in 1920 and 1921 (the second highest award given by the ASCE; it recognizes special commendation as a contributor to engineering science). He urged aid for physical testing because "the methods by which most existing arch dams have been designed are defective and more or less unreliable." Noetzli had been worried about the lack of field data and later that year would publish a paper on tests results in full size dams.[15]

It was becoming clear then that the Service would have to play a major role in the project.[16] In December 1923, W. A. Brackenridge, Senior Vice President of the Southern California Edison Company proposed the building of a large scale concrete arch dam designed expressly for research, and he further offered to provide a large amount of the money for it as well as the use of his company's facilities. Located on Stevenson Creek, a tributary of the San Joaquin River about 60 miles east of Fresno, California, this dam was approved by the committee and construction began in August of 1925.[17]

The test arch design was startlingly thin. The physical structure was set in a V-shaped canyon, and was 60 ft. high with a thickness throughout the top half of only 2 ft. tapering from mid-height to the base from 2 ft. to 7.5 ft. The arch is of a constant 100 ft. radius throughout (see Figure 9).[18] The tests used mechanical strain gages and from these measurements stresses were calculated. Deflection and temperatures were also measured.

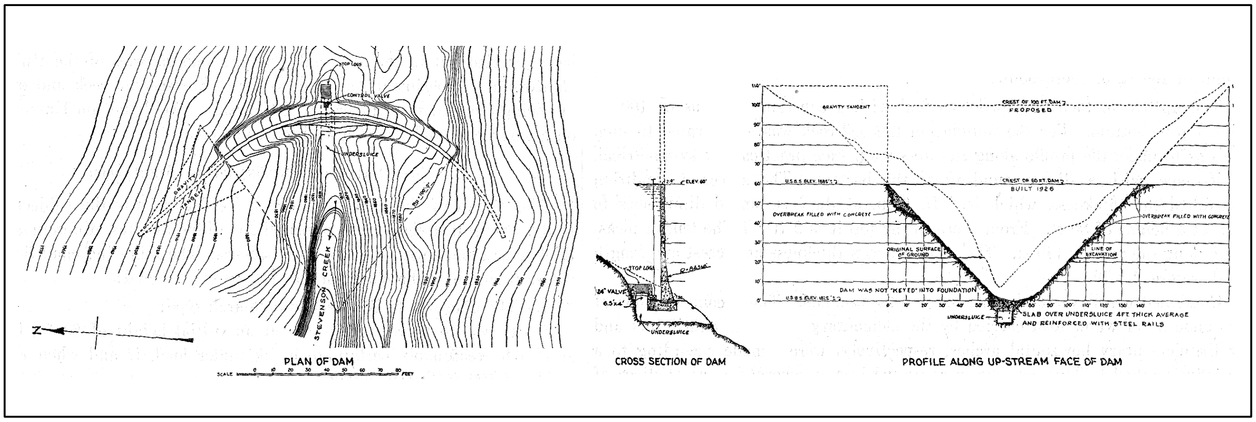


Figure 9 - Stevenson Creek Dam: Plan, section, and profile along upstream face

Meanwhile engineers had been collecting measurements from existing dams as part of the overall program and they had found discouraging results because of the difficulties in relating strains and displacements to loading and temperature changes. They debated the materials from the test dam construction and instrumentation at a meeting in Fresno in early December, 1925.[19]

The Bureau was becoming convinced that the test dam alone would not be sufficient and that a series of small scale models ought also to be included in the program.[20] In early 1926 the Commissioner of the Bureau, Elwood Mead approved funding for part of the work with small scale models.[21] A full report on all this work appeared in November, 1927 and on December 8, 1928 a concrete model of the Stevenson Test Dam was loaded to destruction.[22]

We can summarize the conclusions reached by the committee in late 1927 under three categories: first, the great strength of the arch dams; second, physical experiments have given data useful to engineers developing mathematical analyses; and third, arch dams may be designed more economically (by being thinner) in the future.[23]

The full report included an analysis by Noetzli following his 1921 paper. The results are tabulated in Table 2. An independent crown-cantilever analysis was also carried out by the authors of this paper, based on the procedure outlined in his 1921 paper. The final deflection of arches and crown cantilever under the combined effects of temperature and water pressure are shown in Figure 10. Additionally, a finite element model of the arch under water load was constructed as a comparison to the crown-cantilever method. These results are also tabulated in Table 2, along with actual measurements (excluding temperature effects) taken from the dam itself. As can be seen from the results, the crown-cantilever method overestimates the actual deflections whereas the SAP 2000 ® analysis is relatively close. However, the crown-cantilever method provides a reaonable and conservative approximation of the behavior.

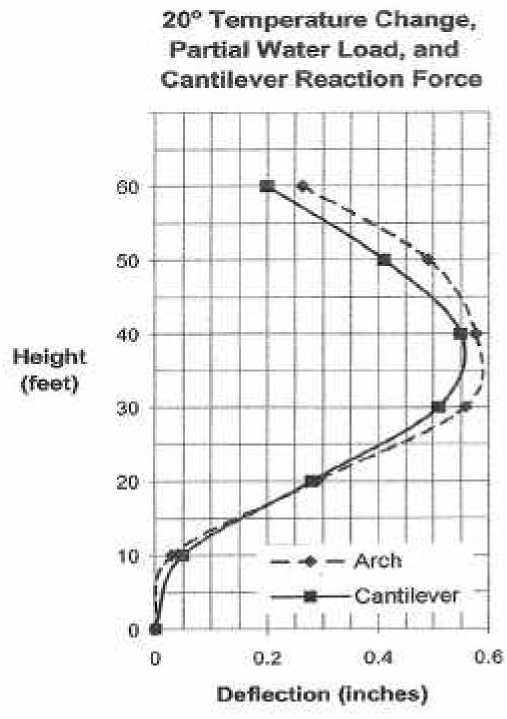


Figure 10 - Deflection of arches and crown cantilever in Stevenson Creek Dam, based on Noetzli's analysis method

Table 2 - Comparison of Stevenson Creek Dam deflections from several analysis methods

	Noetzli's Results	Independent Check of Noetzli's Trial Load Method			SAP 2000 Analysis	Actual measurements from field
		Arch Defl'n				
Elevation	Total (inches)	D_{temp} (inches)	D_{water} (inches)	Total (inches)	Crown Defl'n **(D_{water} only)** (inches)	Crown Defl'n **(D_{water} only)** (inches)
60	**0.254**	0.242	0.022	**0.264**	**0.275**	**0.370**
50	**0.490**	0.240	0.251	**0.491**	**0.345**	**0.385**
40	**0.578**	0.236	0.341	**0.577**	**0.398**	**0.388**
30	**0.559**	0.227	0.332	**0.559**	**0.355**	**0.355**
20	**0.290**	0.165	0.125	**0.290**	**0.183**	**0.234**
10	**0.030**	0.030	0.000	**0.030**	**0.042**	**0.094**
0	**0**	0	0	**0**	**0**	**0**

Hoover and Glen Canyon Dams

During the early planning stages for what became the Boulder Canyon Project, the Director of the Bureau of Reclamation, A. P. Davis, and his staff made an effort to consider a range of possibilities for the design of the big storage dam on the Lower Colorado. Based upon the Service's experiences with the Roosevelt, Elephant Butte, and Arrowrock dams, it is not surprising that a massive concrete/masonry gravity design attracted the interest of Davis, his Chief Engineer Frank Weymouth, Dam Design Engineer John L. Savage, and Project Engineer Walker Young. At the same time, the Service had experience building massive embankment dams (such as Belle Fourche in South Dakota and Strawberry Valley in Utah) as well as thin arch concrete masonry dams (Pathfinder and Shoshone, both in Wyoming); in this context, the decision to utilize a curved gravity concrete design did not come without some consideration of alternative designs. However, the selection did come quickly and without a laborious public review of alternative designs.

These plans were ultimately carried out and the Hoover Dam was designed as a pure gravity dam (see Figure 11), although later the Bureau made a trial load analysis of the structure.

The calculation of the factor of safety against overturning, based on the equations given in Figure 8, shows the considerable overturning resistance of the cross-section. Based on the final dimensions of the dam (height H = 727' and base width B = 660'),

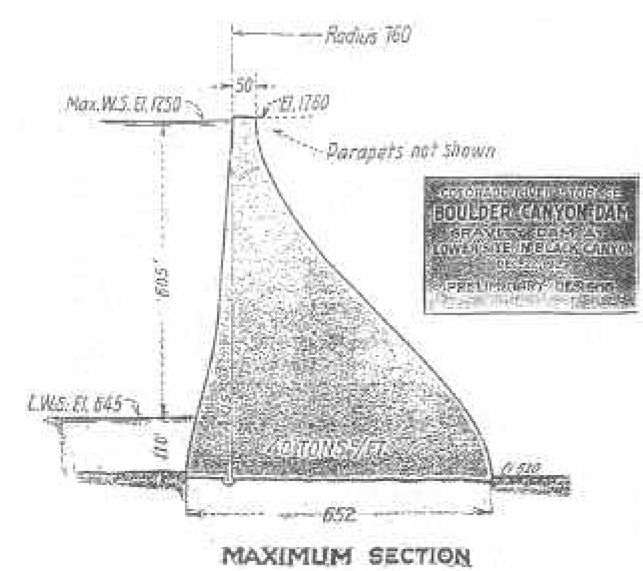


Figure 11 - Hoover Dam. Preliminary design of gravity section.

the factor of safety is $\text{S.F.} = (2.25)(2)\cdot\frac{660^2}{727^2} = 3.7$, neglecting uplift. Considering uplift, the factor of safety decreases to $\text{S.F.} = (1.25)(2)\cdot\frac{660^2}{727^2} = 2$, still reasonably safe. Although the final design utilizes an arched plan, the additional resistance provided by the arching action is unnecessary to carry the loads imposed on the structure. The final structure is shown as built in Figure 12 below.

Unlike Hoover Dam, the Glen Canyon design did benefit from the trial load analysis method that had developed in the 1920s and for the site (located on the main stem of the Colorado River only a few miles upstream from the spot marking the division between upper and lower basins), the Bureau made a design for a thin arched dam. The site had long been familiar to the Bureau. In fact, it had figured as a possible alternative to Boulder/Black Canyon in the early 1920s. By the 1950s, the Bureau was eager to begin construction of a huge 700-foot high dam at Glen Canyon that would represent another major step in the development of the Colorado as a source of hydroelectric power for the burgeoning Southwest. Whereas Echo Park lay within a part of the National Park System and thus comprised a site well suited for wilderness advocates to defend, the Glen Canyon dam and reservoir site simply encompassed federally-owned land and thus was easier to justify in terms of inundating for the greater public good. Although the canyon lands upstream from Glen Canyon could certainly have been characterized as a natural (and national) treasure, they held no place in the national public consciousness and no great movement developed to protect them. Thus, when Congress agreed in 1956 to protect Echo Park, wilderness advocates offered little protest against approval of Glen Canyon Dam in what could later be understood as a de facto compromise regarding development of the two dam and reservoir sites.[24]

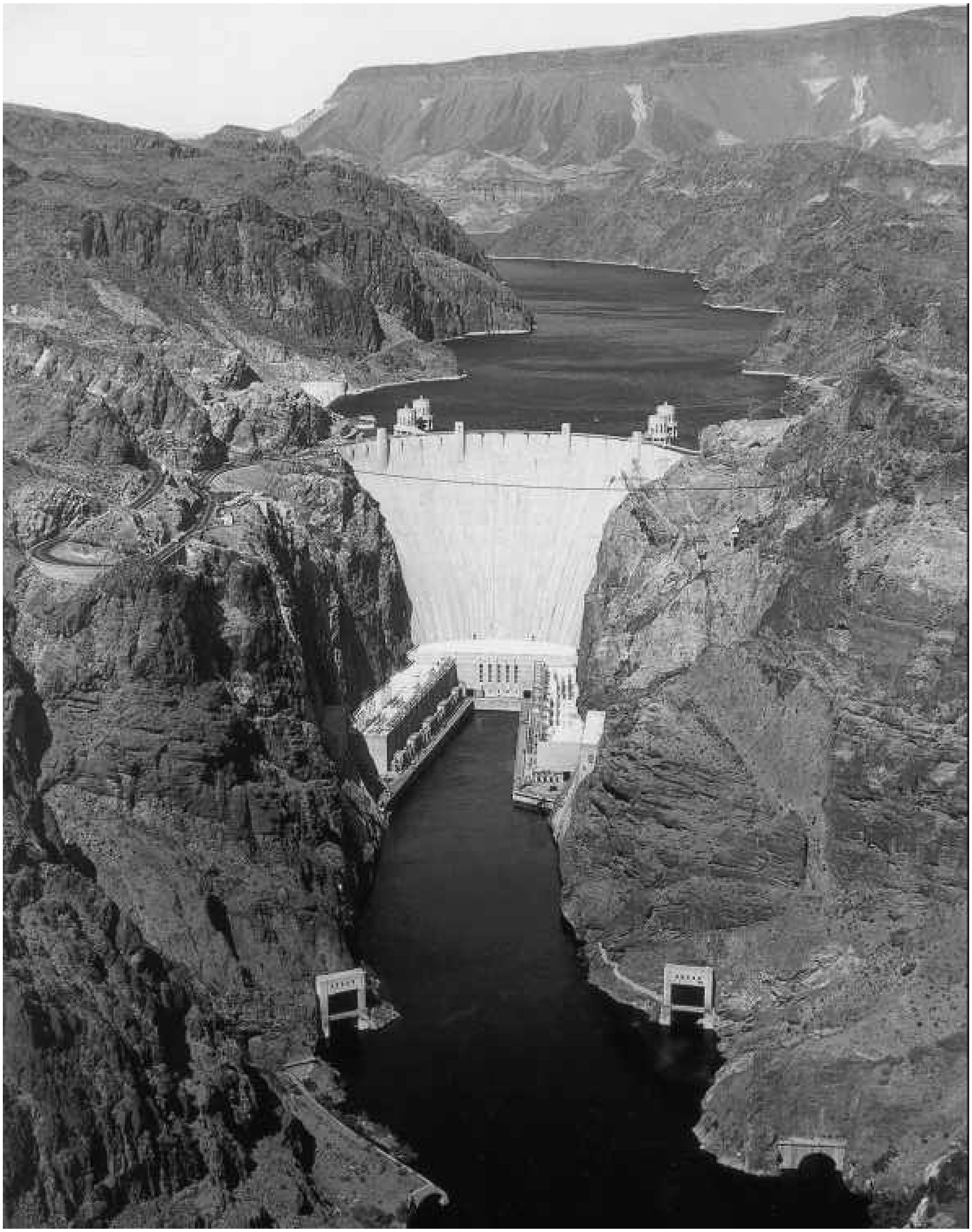


Figure 12 - Hoover Dam: Downstream face showing powerhouses. Source: Bureau of Reclamation.

In terms of design, the Glen Canyon Dam differed from Hoover in its use of an arch design featuring a profile insufficient to stand as a gravity dam; in this strictly technological context it diverged from the precedent set by the Boulder Canyon Project and instead drew from the Bureau's work in building thin arch dams that extended as far back as Pathfinder and Shoshone dams prior to 1910. The final structure (see Figure 13), with a height of 690 ft. and a base thickness of just 290 ft., had a factor of safety against overturning of just 0.80 without uplift and 0.44 with uplift. Thus the design relied heavily on arch action to resist the loads.

The early version of the trial-load analysis, improved on by Noetzli in 1921 and further refined by Bureau engineers in the late 1920's laid a basis for its use on the Glen Canyon Dam.

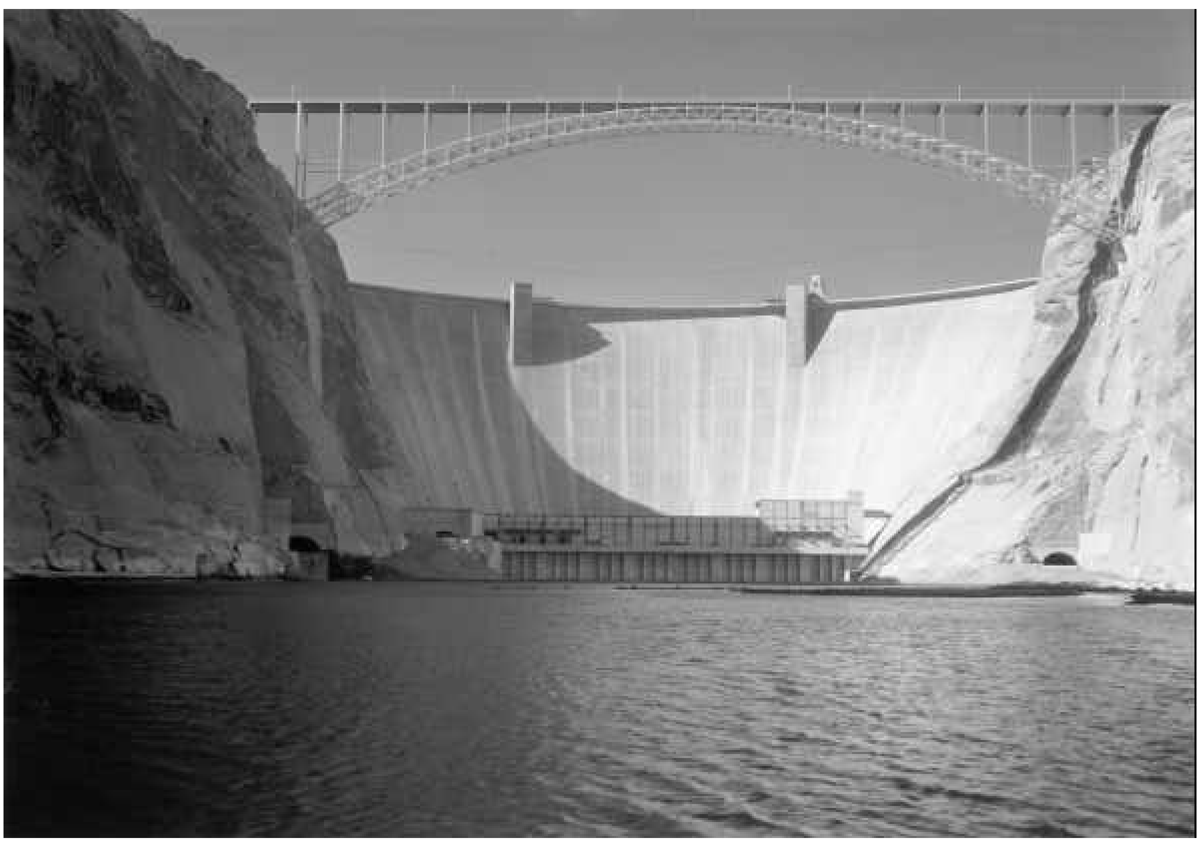


Figure 13 - Glen Canyon Dam. View towards downstream face showing powerhouses. Source: Bureau of Reclamation

This confidence helped lead the design engineers to design and construct a dam far thinner than Hoover and thus rely on arch action instead of only cantilever behavior. There were other reasons too. Concrete quality had improved since the 1920's so that 415 psi stress limit at Hoover could be increased 1000 psi for Glen Canyon.[25]

However, compensating somewhat for the improved concrete, the canyon walls at Glen Canyon were sandstone, a weaker material than the granite walls of Black Canyon. Therefore, the stress at the arches abutments was kept at 600 psi by thickening the arches as they approached the canyon walls. The weaker walls also required the injection of a grout curtain to strengthen the foundations and prevent seepage under and around the dam. These are the hidden dam components that are as essential to safety as the more visually obvious shaped and solidity of the concrete structure itself.

Concrete Forms and Complex Analysis

Fred Noetzli, whose primary aim had been to use tests and calculations, predicted in 1924 that gravity dams would be replaced by thin arch structures; he quoted several engineers saying that “the gravity dam is a thing of the past” and “the gravity dam is an economic crime.” He held the belief common to many in the 1920s that “engineering science is advancing” and that a more rational analytic base would lead to thinner and less costly structures.[26] But Noetzli did not imply that more rational would necessarily mean more complex. He worked with graphical methods typified by his education at the Federal Technical Institute in Zurich. He did not publish the detailed mathematical formulations that had begun to appear in the 1920s and would culminate in the Bureau’s 1938 report.[27]

This type of mathematical complexity was criticized sharply by one of the most famous structural engineering teachers, Hardy Cross of Illinois University. In discussing a highly mathematical 1925 paper on concrete arches, Cross noted the uncertainties of loadings, of actual stress, and of foundations none of which were dealt with in the paper which “having swallowed these ‘camels’ only the ‘gnat’ of mathematical analysis remains. The ‘gnat’ should be an hors d’oeuvre and engineers are giving abnormal gustatory attention to it.” He goes on to proclaim that “the theories of arch analysis which are now being elaborated in engineering literature are distinctly ‘high brow’ in that their elaborateness camouflages with erudition uncertainties and inaccuracies which are inevitable.”[28]

In spite of Noetzli’s hope and Cross’s warning, the profession charged ahead with complexity of analysis and the result was that the Bureau of Reclamation dams did not get thinner but thicker until after World War II when the trial-load analysis would be used to justify the Glen Canyon Dam design. A recent summary of this period stated that:

> “Many arch dams built at the time showed a tendency for increasing thickness. On the one hand the failure of St. Francis Dam in California in 1928 had raised questions regarding the safety of any proposed dam of large size. On the other hand, it seemed that the excellent results obtained at Stevenson Creek, including a verification of the trial load method, were not carried forward with these arch gravity-type dams.”[29]

By 1927 there had emerged well documented traditions of massive and of structural dams. The structural tradition brought forth new methods of analysis both by physical test and by mathematical calculation. The goal had been to build lighter, less expensive, and safer dams. But as the methods of analysis got more complex there seemed to grow an anxiety about uncertainties in the analysis itself and the Federal agencies addressed these worries by designing heavier structures which they believed to be safer even though the lighter ones were performing at least as well.

It seems to be a natural characterization of centralized agencies that they seek to avoid risks, to question innovations, and to justify heavy expenditures by invoking the specter of failure. But behind this apparent criticism, there lay a deep cultural ideology that was characterized by the new and prototypical building material of the 20th century. American society and indeed western society as a whole reacted to reinforced concrete in a profoundly ambiguous way.

Modern concrete clearly stimulated the search for new forms that would carry loads with less material and at least as much safety as heavier designs. But many engineers, not seeing these possibilities or not valuing them, sought to discredit this search for innovation. They saw concrete rather as a mere substitute for stone masonry rather than a new material which, when cast monolithically, made the building of integrated structures possible leading to great savings of materials and weight.

REFERENCES

1. The concept of 'traditions' as applied to dam design is discussed in Donald C. Jackson, *Building the Ultimate Dam: John S. Eastwood and the Control of Water in the West* (Lawrence, Kansas, 1995) pp. 18-21.

2. M. De Sazilly, "Sur un type de profil d'egale resistance propose pour les murs des reservoirs d'eau," *Annales des Ponts et Chaussees* (1853): 191-222; and M. Delocre, "Memoire sur la forme du profil a adopter pour les grande barrs ages en maconnerie des reservoirs," *Annales des Ponts et Chaussee*, Memoires et Documents (1866): 212-272. Both of these references are taken from Norman Smith, *A History of Dams*, Secaucus, N.J., 1972.

3. W. J. M. Rankine, "Report on the Design and Construction of Masonry Dams," *The Engineer* 33 (January 5, 1872): 1-2.

4. When Rankine discussed the significance of the middle third in the 1870s there is no evidence that anyone considered it a reiteration of previously discussed ideas.

5. C.L. Harrison, "Provision for Uplift and Ice Pressure in Designing Masonry Dams", *Transactions ASCE*, Vol 75 (1912), pp. 142-145 with discussion pp. 146-225. For a modern treatment see Max A. M. Herzog *Practical Dam Analysis*, London 1999, p. 80.

6. Wisner reviews this history in his report, George Y. Wisner, "Investigation of Stresses Developed in High Masonry Dams of Short Span." Report submitted to F. H. Newell, Chief Engineer, USGS, Washington, D. C., May 16, 1905, 22 pages.

7. Letter from Wisner et al in Montrose, CA to Newell in Washington, D. C., Oct. 5, 1904; response from Newell to Wisner in Detroit, Oct. 12, 1904; and the recommendation of Wheeler comes in a letter from Wisner and J. H. Quinton to Newell on Nov. 2, 1904. Presumably the final approval came quickly thereafter. For details on A. P. Davis see Charles H. Bissell and F. E. Weymouth, "Memoir for Arthur Powell Davis", *Transactions ASCE*, Vol. 100 (1935) pp. 1582-1591.

8. Letter report from Wheeler in Los Angeles to Wisner in Los Angeles, May 5, 1905, 9 pages. Covering letter from Wisner in Los Angeles to Newell in Washington, D. C. May 16, 1905 with the complete report. The published paper appeared as George Y. Wisner and Edgar T. Wheeler, "Investigation of Stresses in High Masonry Dams of Short Spans", *Engineering News*, Vol. 54, No. 60, Aug. 10, 1905, pp. 141-144. Earlier publications did deal with the arch designs but they appeared in obscure publications; for example, L. Wagoner and H. Vischer, "On the Strains in Curved Masonry Dams". *Proceedings of the Technical Society of the Pacific Coast*, Vol. VI, December 1889. Such articles did not have the influence of Wisner and Wheeler.

9. Fred A. Noetzli, “Gravity and Arch Action in Curved Dams,” *Transactions of the American Society of Civil Engineers*, Vol. 84 (1921), p. 23.

10. Ibid., p. 41. The low factor of safety can be calculated for overturning as follows: For H/B = 1.5 and for no uplift S.F. = 2.00 (assuming a triangular dam section) while for full uplift S.F. = 1.1 (see Figure 3).

11. William Cain, Discussion, Vol. 84, pp. 71-91. His analysis was expanded and published as “The Circular Arch Under Normal Loads,” *Transaction of the American Society of Civil Engineers*, Vol. 85 (1922), pp. 233-248. Noetzli wrote a discussion to the paper in Vol. 85, pp. 261-264.Also in the discussion William Cain, a professor of mathematics at North Carolina University, compared Noetzli’s graphical method to a purely analytic approach given the previous year by B. A. Smith and noted their close correspondence.

12. C. H. Howell and A. C. Jaquith, “Analysis of Arch Dams by the Trial Load Method,” *Transactions of the American Society of Civil Engineers*, Vol. 93 (1929) pp. 1191-1225.

13. R. Wuczkowski, “Flussigkeitsbehälter”. *Handbuch für Eisenbetonbau*, Ed. M von Emperger, Berlin, 1907, Vol 3, pp 348-351, 407-413.

14. Letter from Fred Noetzli in San Francisco to the Board of Trustees of the Engineering Foundation in New York City, March 3, 1922, 2 pages.

15. Fred Noetzli, “Arch Dam Temperature Changes and Deflection Measurements,” *Engineering News-Record*, Vol. 89, No. 22 (Nov. 30, 1922), pp 930-932.

16. Letters F. E. Weymouth in Denver to Davis in Washington, Nov. 10, 1922 and from Morris Bien (acting director) in Washington to Weymouth in Denver, Nov. 20, 1922. In the fall of 1922 Weymouth asked Davis if the Bureau could not give financial support to the project; the acting director agreed to some support.

17. Report on Arch Dam Investigations, Vol. 1, November 1927, *Proceedings of the American Society of Civil Engineers* (May, 1928), pp. 6, 43. In Bulletin No. 2 (June 1, 1924) the Engineering Foundation noted that the Edison Company had put up $25,000 and another $75,000 was needed.

18. A Progress Report on the Stevenson Creek Test Dam, *Bulletin No. 2, Engineering Foundation*, Dec. 1, 1925, 8 pages. On December 1, 1925 the Engineering Foundation issued a progress report giving the dam design and the list of contributors, dominated by private industry. The Bureau of Reclamation is not listed although the Bureau of Standards was, largely through the assignment of W. A. Slater, its engineer-physicist.

19. Letter from Julian Hinds in Denver to R. F. Walter in Denver, Dec. 17, 1925.

20.   Letter from R. F. Walter in Washington to Elwood Mead in Washington, Dec. 16, 1925.

21.   Letters from R. F. Walter in Denver to Elwood Mead in Washington, Feb. 11, 1926, and from Mead in Washington to Walter in Denver, Feb. 19, 1926.

22.   John L. Savage, "Arch Dam Model Tests - Progress Report," Dec. 17, 1928.

23.   Report, Nov. 1927, op. cit, pp. 8-9.

24.   Harvey, *A Symbol of Wilderness*, pp. 280-283 describes how wilderness advocates came to accept the need for Glen Canyon Dam in the mid-1950s; while there was some desire to push for the blocking Glen Canyon Dam it was much less vociferous than the effort to protect Echo Park.

25.   *Glen Canyon Dam and Powerplant*, Bureau of Reclamation, Denver, 1970, pp 55-89.

26.   Fred Noetzli, "Improved Type of Multiple-Arch Dam," *Transactions of the American Society of Civil Engineers*, Vol. 87, 1924, Closure, p. 410.

27. "Trial Load Method of analyzing Arch Dams," Boulder Canyon Project Final Reports, Part V Technical Investigation, *Bulletin 1, Bureau of Reclamation*, Denver CO., 1938.

28.   Hardy Cross, "Discussion of Design of Symmetrical Concrete Arches by C. S. Whitney," *Transactions of the American Society of Civil Engineers*, Vol. 88, 1925, pp. 1075-1077. In 1932 Cross would introduce the most significant simplified analysis procedure ever presented for reinforced concrete frame structures. Noetzli worked from a solid math basis but simplified it by graphic statistics (a Swiss tradition). Cain (ref. 11) was a mathematician, not an engineer, and his work is very complex but Noetzli could understand it easily. The Bureau carried on from Noetzli's simplified approach to develop an elaborate and highly complex mathematical method. Cross criticized all such complexity and sought to develop methods that the average practicing structural engineer could easily use.

29.   Eric Kollgaard and Wallace Chadwick, eds., *Development of Dam Engineering in the United States*, Penguin Press, New York, 1988, p. 269.